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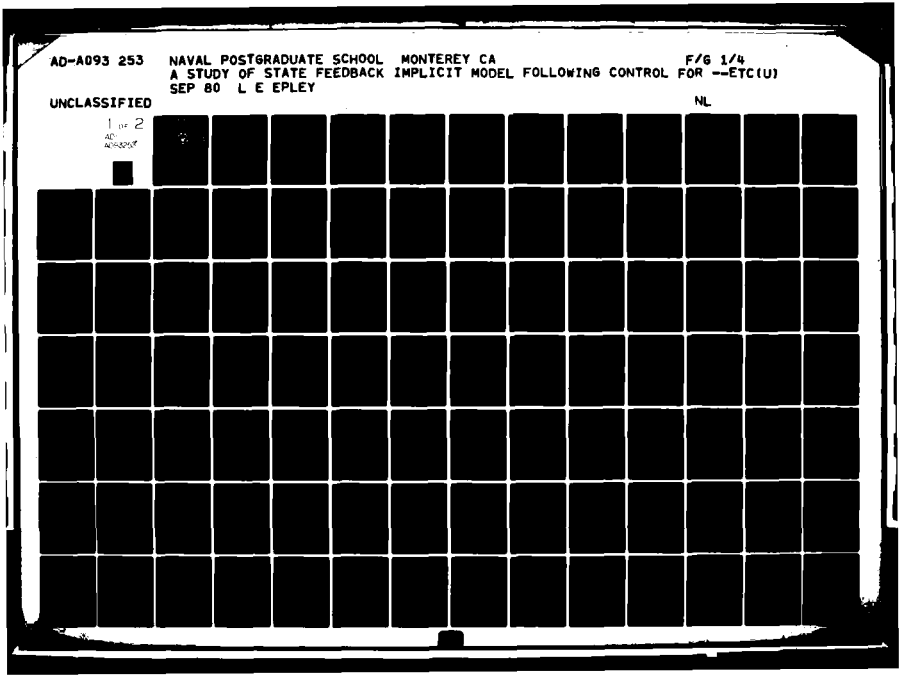

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THESIS

A STUDY OF STATE RATE FEEDBACK IMPLICIT MODEL FOLLOWING CONTROL FOR VSTOL AIRCRAFT

by

Lawrence Ernest Epley

September 1980

Thesis Advisor: D. J. Collins

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A Study of State Rate Feedback Implicit Model Following Control for VSTOL Aircraft

by

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Submitted in partial fulfillment of the requirements for the degree of

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ABSTRACT

The State Rate Feedback Implicit Model Follower control concept is examined in detail from a classical and modern control theory viewpoint. State Rate Feedback Implicit Model Following (SRFIMF) is a concept whereby control of the dynamic response of a system is achieved by the measurement and feedback of a state rate, normally acceleration. In addition to a basic description of the concept, emphasis is placed on the effect of noise in the measurement of the required feedback quantities. Control of the pitch attitude of the AV-8A Harrier VTOL aircraft is used as an example of the application of the control concept. The model of the Harrier used in this study includes the effect of both sensor measurement errors and gust load inputs.

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LIST OF SYMBOLS

A	matrix of coefficients of the closed loop controller
A*	modal transformation of the matrix of coefficients of the closed loop controller
A(s)	SRFIMF controller compensator transfer function
B	matrix of control input coefficients
B*	modal transformation of the matrix of input coefficients
b	damping term in aircraft rigid-body second order modes
c	stiffness term in aircraft rigid-body second order modes
C*	modal transformation of the SRFIMF output matrix
C	SRFIMF output matrix
CPG	constant plant gain or control power gradient depending upon use
F	implicit model following feedback matrix
f	value of the highly damped root of a SRFIMF system
G	matrix of coefficients of the noise inputs to the SRFIMF system
G(s)	Laplace transform of airplane rigid-body modes
H(s)	transfer function representing the dynamic behavior of control force or moment application
h	height in feet of aircraft center of gravity
K	forward loop gain of SRFIMF controller
KRL	SRFIMF controller gain parameter
K_x	position feedback gain
$K_{\dot{x}}$	rate feedback gain

L	matrix of coefficients of the model
L_w	characteristic length of atmospheric turbulance
NASA	National Aeronautics and Space Administration
P	matrix of SRFIMF system covariances
Q	matrix of SRFIMF exponentially corellated white noise strengths
RTA	Research and Technology Aircraft
s	Laplace transform variable
SRFIMF	State Rate Feedback Implicit Model Following
T	transformation matrix between state variable coordinates and modal coordinates
T_r	correlation time of exponentially correlated sensor noises
T_g	correlation time of exponentially correlated wind gust noise
U	Vector of SRFIMF system inputs
V_{wind}	nominal surface wind
W(s)	SRFIMF control law
$W_t(t)$	white noise
X(s)	controlled variable
X	vector of SRFIMF state variables
X*	vector of SRFIMF modes
X_c	commanded input
X_a^*	acceleration error mode
X_p^*	position error mode
X_v^*	velocity error mode
Y	SRFIMF output vector

Z	vector of model states
δ	general transfer function control input
δ_g	gust input
δ_e	control stick input
ζ	second order response damping ratio
θ	pitch attitude angle
μ	white noise strength
ξ	exponentially correlated noise
σ	standard deviation
σ_a	standard deviation of acceleration error
σ_g	standard deviation of gust input
σ_p	standard deviation of position error
σ_v	standard deviation of velocity error
τ	control actuator time constant
ω_n	natural frequency of second order response

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I. INTRODUCTION

A State Rate Feedback Implicit Model Following (SRFIMF) flight controller has been proposed as a possible approach to improving the handling qualities of Vertical and Short Field Take Off and Landing (VSTOL) aircraft by Merrick at the NASA Ames Aeronautical Research Laboratory [1, 2]. The SRFIMF concept has potential applications in various types of control problems encountered in aircraft design, among which are attitude, guidance, and engine control. The concept has not been used in actual flight tests, however several detailed simulations [2, 3] have been conducted at NASA Ames in which SRFIMF control was applied. The most notable features of the control scheme are that:

1. The Input-output relationship of a system using SRFIMF control is insensitive to changes in airframe and propulsion dynamic characteristics.

2. The dynamic relationship of the output to the input is approximately that of a second order system whose frequency and damping is chosen by the designer.

3. The system is self trimming and the commanded output variable is independent of external disturbances.

4. The system has good gust alleviation.

This study presents a detailed analysis of the SRFIMF control concept as applied to the attitude control of VSTOL aircraft.

The following discussion is given to clarify the meaning of State Rate Feedback Implicit Model Following as used in this study. Model following refers to the ability of a control scheme to impart specified dynamic characteristics, given by a model, to the closed loop system. The model being considered here is a second order response in which the parameters of natural frequency and damping ratio are chosen by the designer. Typically a second order response is mathematically defined in terms of the states position and velocity. The state rate of the second order model is the acceleration. The State Rate Feedback Implicit Model Following controller, studied here, achieves model following by measurement and feedback of the system's state rate, acceleration. The result is that a priori knowledge of the plant is not required to produce model following.

To illustrate the use of state rate feedback, consider a plant of second order. With the states of the system X_1 and X_2, defined as position and velocity, the representation of the plant in matrix notation is

$$\begin{Bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{Bmatrix} = \begin{bmatrix} 0 & 1 \\ -c & -b \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} U(t) \qquad 1\text{-}1$$

The acceleration, $\dot{X}_2$, equation is thus

$$\dot{X}_2 = -cX_1 - bX_2 + U(t) \qquad 1\text{-}2$$

The values of b and c define the dynamic behavior of the plant. Design of a control system would in general require that b and c be known. The feedback scheme or control law of the SRFIMF controller is formulated so that the plant dependent quantities of b and c are only involved in the total quantity of $(-cX_1-bX_2)$. With this arrangement the quantity, $(-cX_1-bX_2)$, can be obtained by measurement of the state rate, acceleration, minus the input, U. Model following by measurement of the state rate, as in this illustration, is the basic concept of the SRFIMF controller.

The intent of this study is to provide a detailed analysis of the SRFIMF controller from a modern and classical control viewpoint. Particular emphasis will be placed on a basic description of the control scheme and the effects of measurement errors on the output of the closed loop system. The first two sections deal with a classical analysis of the controller as applied to the attitude position control of a general VSTOL aircraft. The third section considers the effect of measurement errors on the system from a modern control viewpoint. From that analysis, the steady state covariance of the state variables, as a result of measurement uncertainties, will be found. Finally, the previously developed analysis technique will be applied to an example where SRFIMF is used for pitch control of the Harrier aircraft. The effect of sensor errors will be examined and the response of the Harrier to gust inputs will be determined.

The following assumptions are made in this study:

1. The system is linear. Non-linear effects such as control saturation and actuator hysteresis are not considered.

2. The dynamic response of the plant will be represented at a single point by linearized, rigid-body, transfer functions.

3. Measurement uncertainties are represented as exponentially correlated noise and the effect of bias error is not considered. The basic description and development of the SRFIMF controller will be considered in section II.

II. DESCRIPTION OF THE SRFIMF CONTROLLER

A. THE BASIC SCHEME

It is desirable for a flight controller to impart specific dynamic characteristics to the closed loop response of an aircraft system. Often the desired response is given in terms of a natural frequency and damping ratio. Piloted simulations at NASA Ames [2] have indicated that a desirable response from the pilot's stick to the aircraft attitude is a second order response whose natural frequency, ω_n, is approximately 2 rad/sec and damping ratio, ζ, of 0.75. One possible approach to the design of an attitude controller is to apply model following techniques. Mathematically the model for attitude control dynamics can be represented by a transfer function in the frequency domain. If, as an example we let $\theta(s)$ represent the aircraft pitch angle and $\delta(s)$ the elevator control input, then the transfer function of a model for pitch attitude control can be written as

$$\frac{\theta(s)}{\delta(s)} = \frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2} \qquad 2\text{-}1$$

In this example, if the desired response has a natural frequency of 2 rad/sec and damping ratio of 0.75, and equation 2-1 becomes

$$\frac{\theta(s)}{\delta(s)} = \frac{1}{s^2 + 3s + 4} \qquad 2\text{-}2$$

This transfer function will be used to illustrate model following techniques in the development of the SRFIMF concept. The desired model response, given by 2-1, can also be expressed in state variable notation with states Z_1 of position and Z_2 velocity. The matrix of coefficients of the model is given the symbol L and equation 2-1 can be written as

$$\dot{Z} = [L]Z + [B]U$$

$$= \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix} Z + \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} U \qquad 2\text{-}3$$

Given somewhat arbitrary open loop plant dynamics, the object of model following is to produce a closed loop system whose dynamic characteristics are given by equation 2-1 or equation 2-3.

For a general system

$$\dot{X} = AX + BU$$
$$Y = CX \qquad 2\text{-}4$$

the object of implicit model following is to force the output of the system to follow the model equation

$$\dot{Z} = LZ + BU \qquad 2\text{-}5$$

That is to say, the output, Y, should approximate Z ($Y \approx Z$) so that

$$\dot{Y} \approx LY + BU \qquad 2\text{-}6$$

By proper choice of the feedback gain F, as shown in figure 2-1, the output is forced to follow equation 2-6. The

implied model shown in the upper portion of the figure, is not actually generated but implied by the behavior of Y.

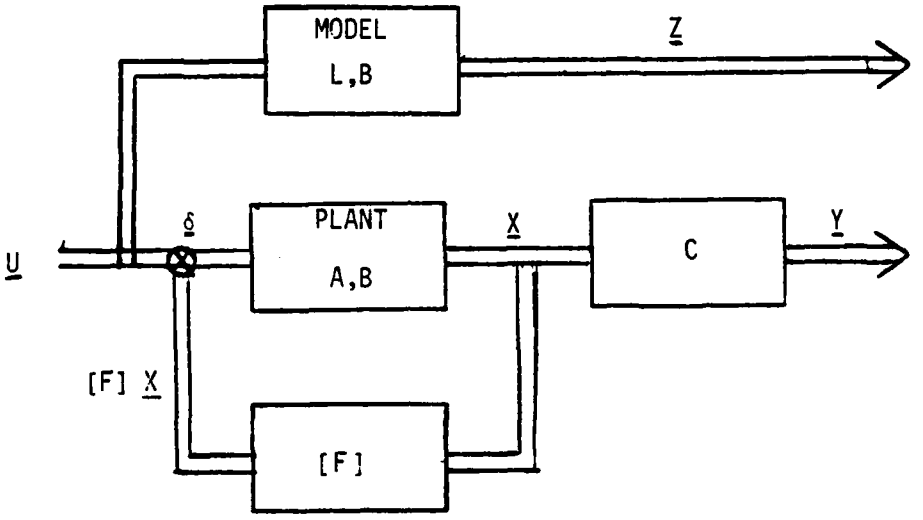

Figure 2-1 Implicit Model Following scheme

The formulation of the feedback law required in model following has been determined by Taylor [4] using optimal control techniques. In addition, Erzberger [5] has defined algebraic methods for determining if perfect model following can be achieved. This algorithm will be examined later in this section. In aircraft applications certain physical facts about the system allow a form of implicit model following to be obtained by simple intuitive reasoning.

Consider the problem of stabilizing the pitch attitude of a VSTOL aircraft. The following assumptions can be made and will lead to a simplified SRFIMF controller:

1. The desired model response is of second order with frequency, ω_n, of 2 rad/sec and damping ratio, ζ, of 0.75 from equation 2-1.

2. Measurements of angular position, angular rate and angular acceleration are available.

3. The control U is either a thrust or control deflection whose net result is to produce angular acceleration of the vehicle.

4. The open loop plant is arbitrary and the transfer function for θ, $\theta(s)/U(s) = G(s)$, may be unknown.

5. The control law which produces model following will be developed so that it represents the difference between the vehicle's angular acceleration, as measured, and the angular acceleration which would be implied by the model given in 1, above.

From assumption 1 we obtain the desired closed loop response as that of the model and express it as

$$\frac{\theta(s)}{U(s)} = \frac{1}{s^2 + K_x^{\bullet}s + K_x} \qquad 2\text{-}7$$

where $\theta(s)$ is the pitch attitude and the constants K_x and $K_x^{\bullet}$ are defined for convenience and will be used throughout this study as

$$K_x^{\bullet} = 2\zeta\omega_n = 3 \qquad 2\text{-}8$$

$$K_x = \omega_n^2 = 4 \qquad 2\text{-}9$$

By requiring the model, equation 2-7, to hold we observe that the implied angular acceleration is

$$s^2\theta(s) = -K_x\theta(s) - K_{\dot{x}}s\theta(s) + U(s) \qquad 2\text{-}10$$

$\theta(s)$ and $s\theta(s)$ are angular position and angular rates which, by assumption 2, are available from measurements. The quantity $U(s)$ is the input to the open loop plant. Given $\theta(s)$, $s\theta(s)$ and $U(s)$, the quantity $s^2\theta(s)$ can be calculated from equation 2-10.

We will define $W(s)$ as the control law for the system. $W(s)$ will be taken as the difference between the implied model acceleration $s^2\theta(s)$ and the actual measured acceleration

$$W(s) = s^2\theta(s) - (\text{measured acceleration}) \qquad 2\text{-}11$$

Using equation 2-10 and substituting for $s^2\theta(s)$ one has

$$W(s) = -K_x\theta(s) - K_{\dot{x}}s\theta(s) + U(s) - (\text{measured acceleration}) \qquad 2\text{-}12$$

Further the quantities $\theta(s)$ and $s\theta(s)$ will be determined by sensors so that

$$W(s) = -K_x \cdot (\text{measured position}) - K_{\dot{x}} \cdot (\text{measured rate}) + U(s) - (\text{measured angular acceleration}) \qquad 2\text{-}13$$

The quantity U(s) could be considered pilot or other control input to the plant. Since we also defined W(s) as the control there is some ambiguity in the notation. The symbol U(s) will be used consistently for the control feedback quantity as indicated in figure 2-2.

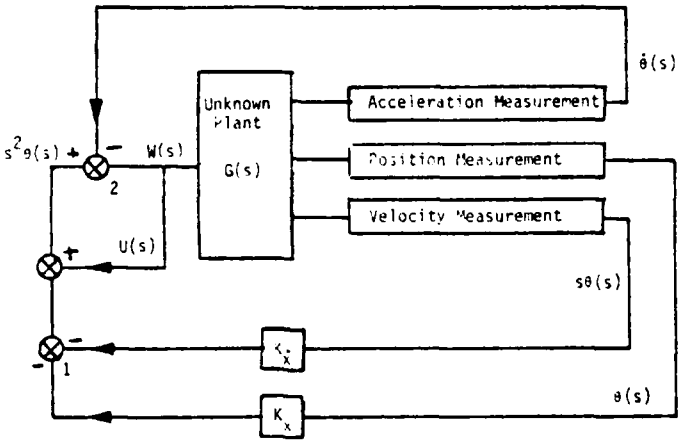

Figure 2-2 SRFIMF position controller developed by intuitive arguments.

Figure 2-2 shows schematically the control law and unknown plant G(s). It can also be seen that the plant input, U(s) is a feedback quantity and the implied acceleration, $s^2\theta(s)$ is compared to the measured acceleration at point 2 of the figure. The term state rate feedback is applied to this type of control because of acceleration feedback. It is this measurement which brings to the controller information about the plant making a priori knowledge of the transfer function for $\theta(s)$ unnecessary. This aspect of the SRFIMF controller is most clearly seen by application of the Erzberger criterion mentioned earlier.

The Erzberger criterion for exact implicit model following is obtained by analysis of the system in state space representation where the system is given by

$$\dot{X} = AX + BU \qquad 2\text{-}14$$

$$Y = CX \qquad 2\text{-}15$$

For implicit model following the output is approximated by

$$\dot{Y} = LY \qquad 2\text{-}16$$

Taking the derivative of equation 2-15, one has

$$\dot{Y} = C\dot{X} \qquad 2\text{-}17$$

Substituting from equation 2-14

$$\dot{Y} = C(AX + BU)$$

or

$$\dot{Y} = CAX + CBU \qquad 2\text{-}18$$

but from equations 2-15 and 2-16 one has

$$\dot{Y} = LY = LCX \qquad 2\text{-}19$$

so that one has, on equating 2-18 and 2-19,

$$CAX + CBU = LCX$$

or

$$CBU = (LC - CA) \cdot X \qquad 2\text{-}20$$

Solving for the control, U, from 2-20 we have that

$$U = \left[(CB)^{+} \cdot (LC - CA)\right] X \qquad 2\text{-}21$$

where $(CB)^{+}$ is the pseudo, or generalized, inverse of (CB). Eliminating U from 2-20 and 2-21, the condition for perfect model following becomes

$$[(CB)(CB)^{+} - I][(LC) - (CA)] \cdot X = 0 \qquad 2\text{-}22$$

The use of the pseudo inverse is based on the property of the pseudo inverse that $(CB)(CB)^{+}$ is an orthogonal projection operator on the range of (CB). It then follows that if 2-22 holds for all X, the range of CB must contain the range of (LC-CA). This implies that 2-16 is valid which has already been assumed to be the case.

We will now apply equation 2-22 to the pitch attitude controller given earlier. In this case $\theta(s)/U(s)$ is assumed to be the transfer function of an arbitrary second order system.

$$\frac{\theta(s)}{U(s)} = \frac{1}{s^2 + bs + c}$$

The desired closed loop performance is given by the model as in equation 2-7. For this example we initially take the feedback quantities to be angular position and angular rate. Defining the quantities in state variable representation we have the plant where

$$X_1 = \theta(t)$$

$$X_2 = \dot{\theta}(t)$$

$$\dot{X} = [A]\ X + BU$$

$$= \begin{bmatrix} 0 & 1 \\ -c & -b \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} \cdot \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} U \qquad 2\text{-}23$$

and

$$Y = CX$$

or

$$Y = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} \qquad 2\text{-}24$$

Calculation of the pseudo inverse depends upon the relative rank of C and B in this case, following Noble [6], the pseudo inverse of (CB) is given as

$$(CB)^+ = [(CB)^+ \cdot (CB)]^{-1} \cdot (CB)^T$$

From 2-23 and 2-24

$$(CB) = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix}$$

therefore

$$(CB)^+ = [0\ 1]$$

To determine if perfect model following is possible, we substitute into 2-22 using 2-23, 2-24, 2-8, 2-9 and the above for (CB) and $(CB)^+$

$$[(CB)\ (CB)^{+} - I] = \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix} \qquad 2\text{-}25$$

$$[(LC) - (CA)] = \begin{bmatrix} 0 & 0 \\ -(K_x + c) & -(K_{\dot{x}} + b) \end{bmatrix} \qquad 2\text{-}26$$

The result is that

$$[(CB)(CB)^{+} - I]\ [(LC) - (CA)] \equiv 0 \qquad 2\text{-}27$$

Equation 2-27 shows that perfect model following is possible for all X using position and rate feedback only. The control can be determined by equation 2-21 with the result

$$U_c = [(-K_x + c)(-K_{\dot{x}} + b)] \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} \qquad 2\text{-}28$$

$$= -K_x X_1 - K_{\dot{x}} X_2 + cX_1 + bX_2 \qquad 2\text{-}29$$

The control law given in equation 2-29 requires that the constants c and b be known in order to produce the desired model following. In the previous discussion we stated that the addition of acceleration feedback provided the needed information about the plant. To show this, note that the latter two terms of equation 2-29, $cX_1 + bX_2$, can be interpreted in terms of the acceleration of the system, X_2, since

$$\dot{X}_2 = -cX_1 - bX_2 + U(s) \qquad 2\text{-}30$$

one has

$$\dot{X}_2 - U(s) = -cX_1 + -bX_2 \qquad 2\text{-}31$$

or rewriting equation 2-29 one has

$$U_c = -K_x X_1 - K_{\dot{x}} X_2 + U(s) - \dot{X}_2 \qquad 2\text{-}32$$

We can compare this to equation 2-13 rewritten in the same notation

$$W(s) = -K_x X_1 - K_{\dot{x}} X_2 + U(s) - (\text{measured angular acceleration}) \qquad 2\text{-}33$$

Note, X_1 and X_2 are the measured angular position and measured angular rate. This comparison shows that the measured acceleration supplies the terms needed for perfect model following without a need for a knowledge of the plant dynamics.

The significance of this result is that state rate feedback can be used to provide information about the plant in applications where the plant has unknown and changing dynamic characteristics. This conclusion was reached based upon the assumption that the plant was of second order. We shall now consider a more general form of the SRFIMF controller and we will show that model following can be achieved by measurement of state rate for a higher order plant.

B. THE GENERAL FORM OF THE SRFIMF CONTROLLER

The preceding discussion presented an intuitive description of the principal operation of the SRFIMF controller and the use of state rate feedback. It is the intent of this section to develop a general form of state rate feedback and

to show again that model following can be achieved without a priori knowledge of the plant by using measurement of the state rate. We begin by examining the basic SRFIMF controller as developed by Merrick at NASA Ames. The block diagram of this controller is shown in figure 2-3.

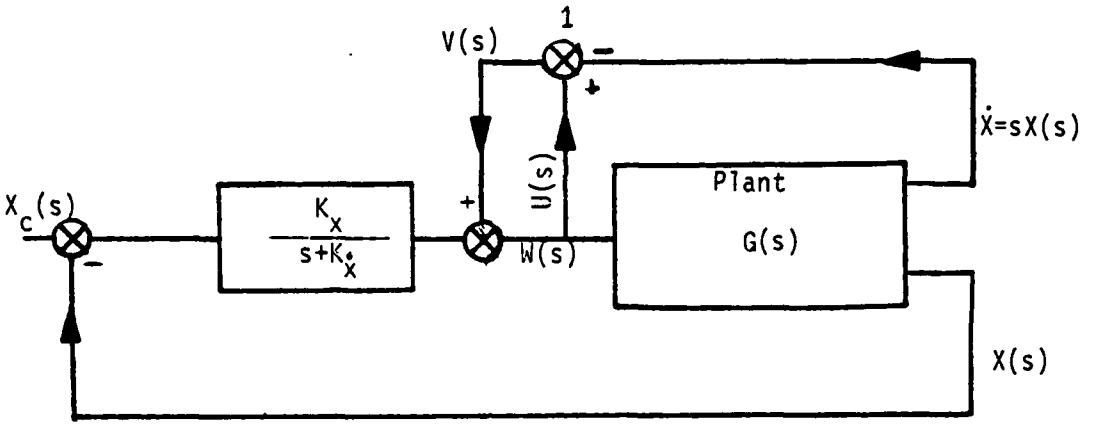

Figure 2-3 The general form of the SRFIMF controller.

Assume that figure 2-3 represents a velocity controller, then the feedback quantities X(s) and sX(s) are velocity and acceleration respectively. It can be seen that at position 1 we are summing (- acceleration + U(s)). It will be shown that the transfer function between the input $X_c(s)$ and the output X(s) does not depend upon the plant transfer function G(s) and that the closed loop response is that of a second order system whose damping and natural frequency are determined by the choice of feedback constants K_x and $K_{\dot{x}}$.

From figure 2-3 W(s) is given by

$$W(s) = U(s) - sX(s) + [X_c(s) - X(s)] \cdot \left[\frac{K_x}{s + K_x^{\cdot}}\right] \quad 2\text{-}34$$

and for V(s) we write

$$V(s) = U(s) - sX(s) \quad 2\text{-}35$$

from the definition of the transfer function G(s) we have that

$$X(s) = W(s)\ G(s) \quad 2\text{-}36$$

Combining 2-34 and 2-36 we have that

$$X(s) = \left\{ U(s) - sX(s) + \frac{(X_c(s) - X(s))\ K_x}{s + K_x^{\cdot}} \right\} G(s) \quad 2\text{-}37$$

Since U(s) = W(s) as seen from figure 2-3, we have

$$X(s) = U(s) \cdot G(s) \quad 2\text{-}38$$

and equation 2-37 can be rewritten as

$$X(s) = X(s) - sX(s)G(s) + \frac{(X_c(s) - X(s))\ K_x}{s + K_x^{\cdot}}\ G(s) \quad 2\text{-}39$$

and

$$sX(s)G(s) = \frac{(X_c(s) - X(s))\ K_x}{s + K_x^{\cdot}} \cdot G(s) \quad 2\text{-}40$$

G(s) on both sides of equation 2-40 cancels and the result becomes

$$\frac{X(s)}{X_c(s)} = \frac{K_x}{s^2 + K_x^{\cdot}s + K_x} \quad 2\text{-}41$$

Equation 2-41 shows that the closed loop response of the SRFIMF is identically that of a second order system whose natural frequency and damping ratio is completely determined by the constant feedback gains which are given by

$$\omega_n = K_x \qquad \zeta = \frac{K_{\dot{x}}}{2\sqrt{K_x}}$$

The results of this analysis indicate that the SRFIMF controller is an excellent candidate for VSTOL aircraft applications. The form of the controller shown in figure 2-2 could be used to control a position such as attitude while the general form shown in figure 2-3 might be used to control rates or velocity. It will now be necessary to examine the requirements necessary to impliment a SRFIMF controller in an aircraft. In particular, the exact relationship between U(s) and W(s) must be considered. The next section will examine practical examples of SRFIMF controllers in a realistic aircraft environment.

C. PRACTICAL EXAMPLES OF THE SRFIMF CONTROLLER

Two types of controllers are illustrated in this section, a position controller, figure 2-4, and a velocity controller, figure 2-5. A first order actuator is included in the plant of each controller. Also included is a compensator in the control feedback loop involving U(s). The system is not realizable without the compensator. These controllers will be used in the analysis of this report.

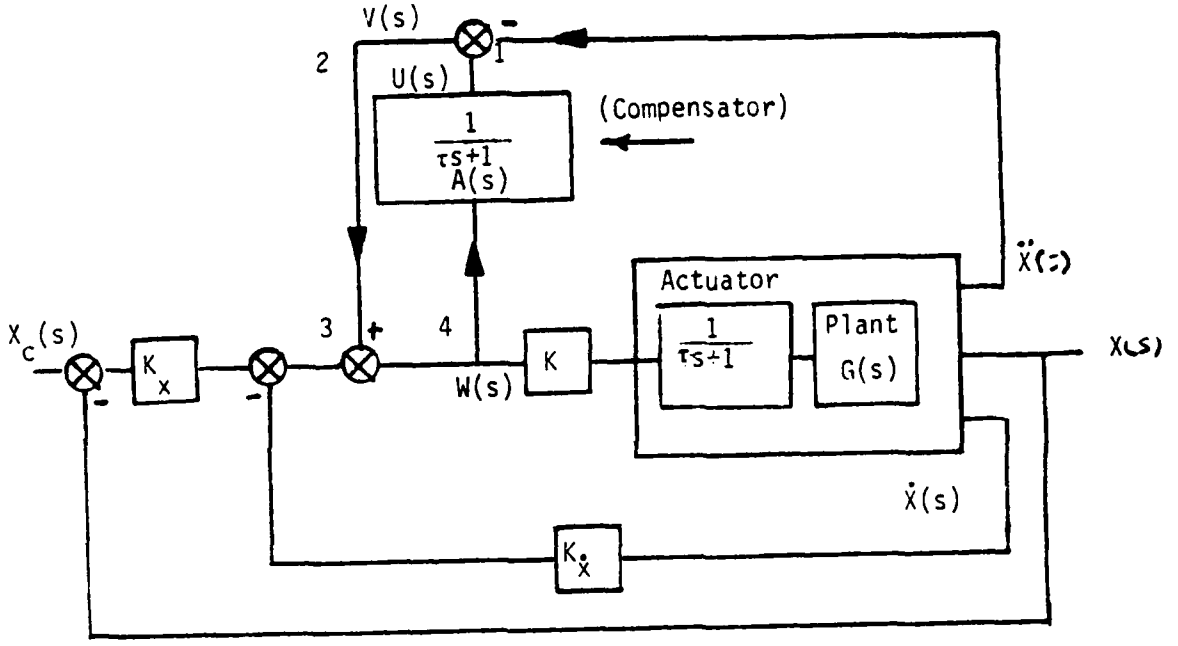

Figure 2-4 Physically realizable SRFIMF position controller.

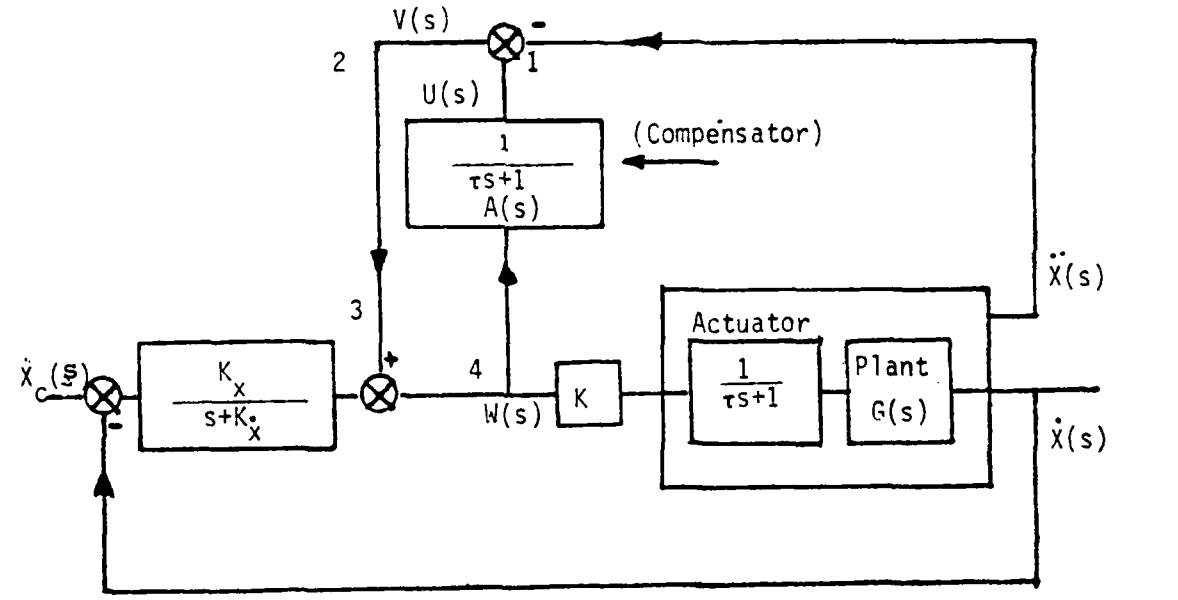

Figure 2-5 Physically realizable SRFIMF rate controller.

The plant is assumed to be driven by a control actuator and because of that a phase lag can be expected to exist between the commanded control signal, W(t) and the input of the plant. The effect of the actuator will be modeled by using the first order transfer function

$$\frac{H(s)}{W(s)} = \frac{1}{(\tau s + 1)} \qquad 2\text{-}42$$

where H(s) is the output of the actuator, W(s) is the input and τ is the actuator time constant assumed to be 0.1 sec for all of the examples considered in this work. Recall from a previous section that SRFIMF feedback contained a term U(t) which represented the control input to the plant and that W(t) was assumed to equal U(t). Because of the fact that U(t) was equal to W(t) the output signal of the controller could be used to cancel the U(t) term in the acceleration feedback. It was shown that

$$\text{acceleration} = -cX_1 - bX_2 + U(t) \qquad 2\text{-}43$$

and it was assumed that

$$U(t) = W(t) \qquad 2\text{-}44$$

In order for equation 2-44 to be valid, the system would need to have instantaneous response to any input. Because of the action of the control actuator this is not possible and a compensator is placed in the feedback loop between W(s) and the summing junction at 1, as shown in figures 2-4 and 2-5. The compensator transfer function is A(s). A(s) is chosen so that the output of the compensator is dynamically identical to the input of the plant, after the control actuator. When the control is represented by a first order lag, as in this case, A(s) is the same as the transfer function of the actuator. In cases of higher order actuator dynamics the result is more complicated. An algorithm for determining the necessary transfer function, A(s), which must be used in the feedback is given by Merrick [2]. We will assume a first order actuator and a compensator of the form $1/(\tau s + 1)$ as shown in figures 2-4 and 2-5, for the remainder of this work.

Actual controllers used at NASA Ames are illustrated in figures 2-6 and 2-7. Figure 2-6 is a speed controller. The quantities VD and VDD represent the measured speed and accel-acceleration. VC is the commanded speed and W(s) is the output of the controller. Figure 2-7 is a SRFIMF position controller used to control the pitch attitude, θ, of the RTA vehicle. The limiters seen in the figures are not included in the model studied here. The purpose of the limiter is to prevent the control feedback loop from acting as an integrater when the plan control is saturated. This condition could occur

when a large difference between the commanded variable, X, and the input variable, X_c, existed and the plant control was saturated. In this condition W(s) would increase without bound and a reversal of the input would be delayed because of the very high value of W(t) at the time the control reversal was applied. The limiter was not considered here because of the assumption of linearlity.

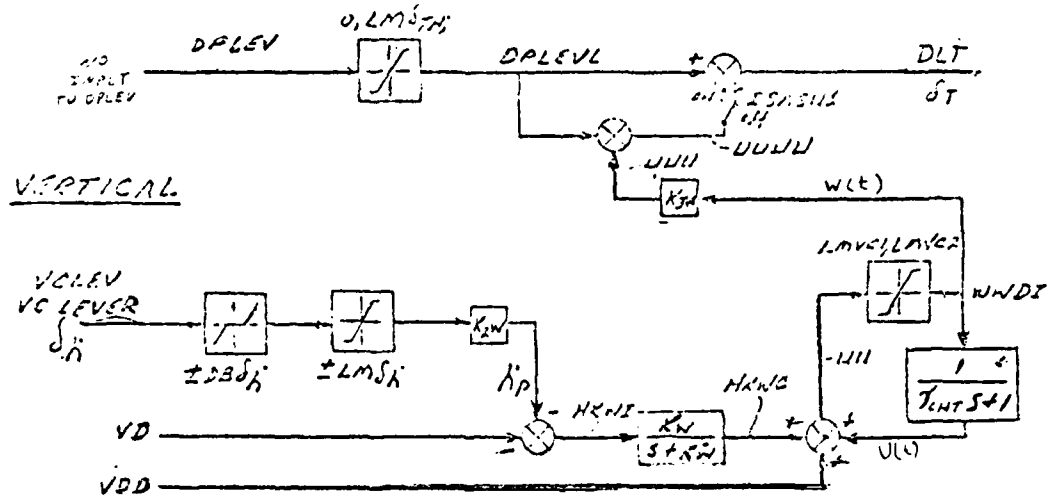

Figure 2-6 SRFIMF controller employed on the RTA for speed control.

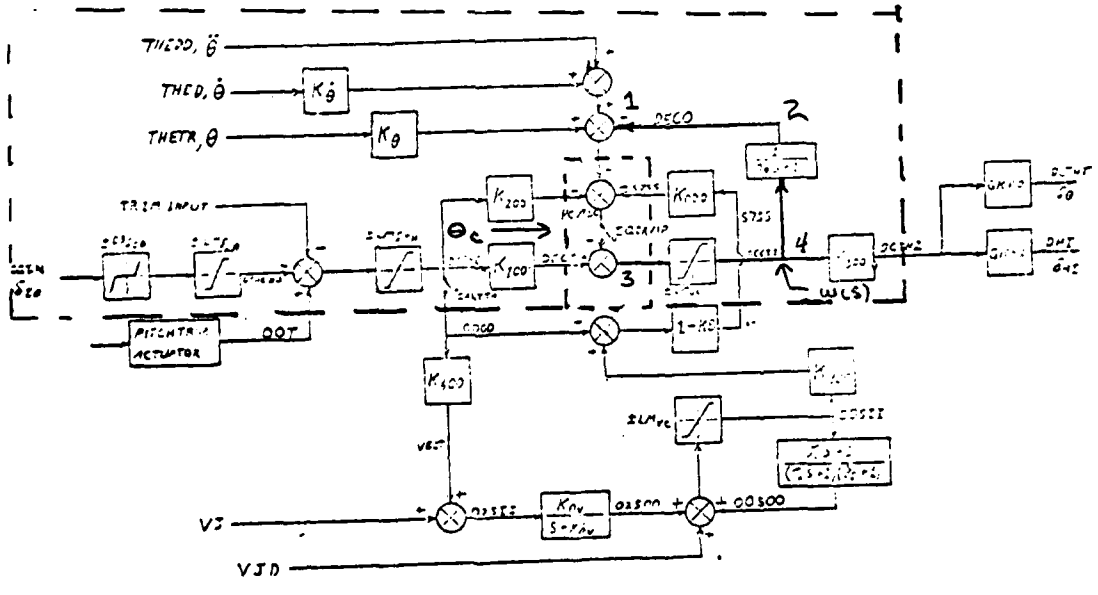

Figure 2-7 SRFIMF controller employed on RTA for pitch attitude control.

III. TIME AND FREQUENCY DOMAIN ANALYSIS OF THE SRFIMF

A. REPRESENTATIVE AIRCRAFT PLANT DYNAMICS

The State Rate Feedback Implicit Model Follower controller presented in section II, figure 2-4 will now be examined using Root Locus, Bode Analysis and Time Simulations. Root Locus analysis will show the asymptotic behavior of the closed loop poles and Bode analysis will show the filter characteristics of the controller. Simulated time response of the controller will be shown in order to demonstrate the ability of the SRFIMF to follow the given model and to show graphically the self trimming feature of the controller. The model and representative plant will next be defined.

The model for the system is taken as a second order system with a transfer function given by

$$\frac{M(s)}{U(s)} = \frac{K_x}{s^2 + K_{\dot{x}}s + K_x} \qquad 3\text{-}1$$

The gains K_x and K_x are selected to yield two different models, one a position controller and the other a rate controller as shown in the following table.

	ω_n	ζ	K_x	$K_{\dot{x}}$	Pole Location	
Attitude Control	2 rad/sec	.75	4	3	-1.5	1.32i
Rate Control	1.23 rad/	.7023	1.57	1.76	-.88	.89i

TABLE 3-1 Assumed Model Constants.

The values listed in table 3-1 are based upon piloted simulations at NASA Ames [2] and reflect what has been judged to be representative of good handling qualities.

The plant transfer function is also taken as a second order system

$$G(s) = \frac{CPG}{s^2 + bs + c} \qquad 3\text{-}2$$

While initially this assumption may seem unrepresentative of a real aircraft, proper choice of b and c can represent the dynamic behavior of the individual modes of any aircraft, provided that first order modes are taken two at a time. To illustrate, consider the general transfer function

$$C(s) = \frac{K \prod_{i=1}^{M} (s + Z_i)}{s \prod_{J=1}^{q} (s + P_J) \prod_{K=1}^{r} (s^2 + 2\zeta_K\omega_K s + \omega_K^2)} \qquad 3\text{-}3$$

If the poles are distinct then equation 3-3 can be expanded into partial fractions as follows

$$C(s) = \frac{a}{s} + \sum_{J=1}^{q} \frac{a_J}{s + P_J} + \sum_{K=1}^{r} \frac{b_K(s + \zeta_K\omega_K) + C_K W_n \sqrt{1-\zeta_K^2}}{s^2 + 2\zeta_K\omega_K s + \omega_K^2} \qquad 3\text{-}4$$

It can be seen from the above that the response of a higher order system is composed of a summation of first and second order terms. When the first order terms are taken two at a

time, the total system response can be expressed as the sum of only second order terms with constant numerators. The assumption of linearity allows us to analyze the modes independently. It remains to choose values of b and c which represent typical aircraft modes. To do this, it will be necessary to survey several representative aircraft. From this survey a number of modes will be selected as typical of VSTOL applications. It will be assumed that acceptable control of each of the modes will lead to acceptable control of the system. This method of analysis leads to considerable simplification since it allows the plant to be taken as a second order system. While this simplified plant will serve the majority of the analysis, a more complex plant will be examined in section V.

Table 3-2 is a listing of pole locations for selected aircraft. Figure 3-1 is a sketch of these poles in the complex plane and an assumed envelope of VSTOL pole locations. Table 3-3 lists points which will be considered by time simulation. From this list, four representative modes will be examined by root locus.

AIRCRAFT	FLIGHT CONDITION	FREQUENCY/DAMPING		OSCILLATORY POLES		REAL POLES	
		ω_n	ζ	Re.	Im.	$1/T_1$	$1/T_2$
F106	S.L. M=.2	2.42	.62	-1.5	1.9	-.169	-.59
Lateral/Directional	20,000', M=.9	3.01	.159	-.48	3.0	-1.84	+.006
A4	S.L. M=.2						
Longitudinal	short period	1.56	.31	-.48	1.5		
	long period	1.52	.087	-.013	.15		
	15,000', M=.9						
	short period	.623	.344	-.20	.585		
	long period	.12	-.078	+.01	.12		
A4	S.L. M=.2	1.89	.05	-.09	1.89	-.065	-.56
Lateral/Directional	15,000', M=.9	6.61	.096	-.641	6.6	-2.48	-.006
VZ4	0 kts.	.731	-.439	+.32	.66	-.82	-.137
Longitudinal	26.5 kts.	2.16	.4	-.086	.20		
	75.6 kts.						
	short period	3.4	.374	-.127	3.4		
	long period	.316	.346	-.11	.30		
VZ4	0 kts.	.669	-.347	+.30	.63	-.65	-.90
Lateral/Directional	75 kts.	1.59	.421	-.67	1.4		
H13	0 kts.	.43	-.250	+.10	.42	-.69	-.87
(helo)	115 kts.	.38	-.043	+.016	.40	-.9	-1.05
Harrier	0 kts.	.31	-.48	+.148	.27	-.33	-.02
Longitudinal	60 kts.	.32	-.91	+.30	.13	-.073	-1.0
Harrier	0 kts.	.52	-.50	+.26	.45	-.015	-.58
Lateral/Directional	60 kts.	1.2	-.28	+.336	1.15	-.068	-1.26
	120 kts.	1.8	-.15	+.27	1.78	-.056	-1.73

Table 3-2 Typical aircraft pole locations where the characteristic equation has the form

$$(s + 1/T_1)(s + 1/T_2)(s^2 + 2\zeta\omega_n s + \omega_n^2) = 0$$

Source: References [4 & 6]

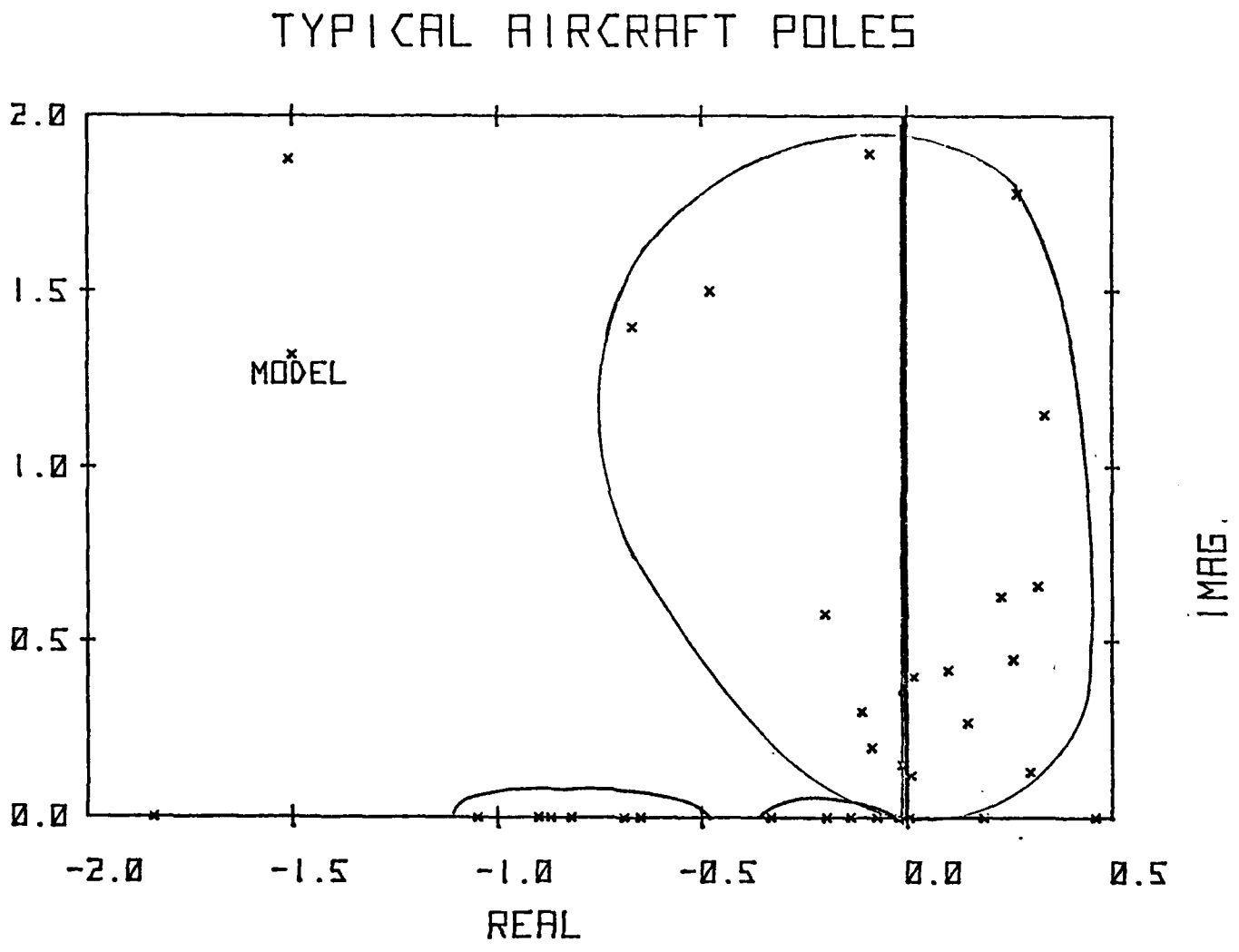

Figure 3-1 Typical aircraft pole locations.

Mode #	Real	Im.	b	c
1	-.6	3.	1.2	8.64
2	-.15	2.15	0.3	6.23
3	-.3	1.0	0.6	0.91
4	-0.1	0.3	0.2	0.08
5	-0.2/0.		0.2	0.0
6	0.3	1.5	-0.6	2.16
7	0.3	0.0	-0.6	0.0
8	0.1	0.14	-0.2	0.03
9	0.0	0.0	0.0	0.0
10	-1.0/ 0.6		1.6	0.6

Table 3-3 Aircraft modes used in simulations. The modes were choosen to be typical of VSTOL aircraft as seen in figure 3-1.

B. FREQUENCY RESPONSE AND BODE ANALYSIS

The object of Bode Analysis is to examine the frequency response characteristics of the SRFIMF controller. It will be necessary to develop the transfer function of the position controller shown in figure 2-4. The plant transfer function is given in general as

$$G(s) = \frac{X(s)}{W(s)} = \frac{CPG}{s^2 + bs + C}$$

As in earlier sections the control actuator transfer function, H(s), is assumed to be of the form

$$H(s) = \frac{1}{(\tau s + 1)} \qquad 3\text{-}5$$

The transfer function of the plant is given by G(s), the output position, defined as X(s) and the commanded input is $X_c(s)$. The compensator shown in figure 2-4, and described in section 2-3 is

$$A(s) = \frac{1}{(\tau s + 1)} \qquad 3\text{-}6$$

From figure 2-4 we can write at point 1, the equation

$$V(s) = A(s)W(s) - s^2X(s)$$

where $s^2X(s)$ is now the acceleration feedback. We write the control quantity, W(s), from figure 2-4 as

$$W(s) = U(s) - K_x sX(s) - K_x [-X(s) - X_c(s)] \qquad 3\text{-}7$$

and also note

$$X(s) = K \cdot G(s) \cdot H(s) \cdot W(s) \qquad 3\text{-}8$$

Combining and rearranging the above expressions we have that

$$X_c(s)K_x = \left(\frac{(1 - A(s))}{K} \cdot \frac{1}{G(s)H(s)} + s^2 + K_x s + K_x \right) \cdot X(s)$$

and further rearrangement leads to the transfer function for the input-output relation

$$\frac{X(s)}{X_c(s)} = \frac{K_x}{\left(\left[\frac{1-A(s)}{K} \right] \cdot \frac{1}{G(s)\ H(s)} \right) + s^2 + K_x s + K_x} \qquad 3\text{-}9$$

It will be convenient to separate the constant plant gain, CPG, from the plant transfer function. The remaining transfer function, defined as G'(s), has a unity constant gain. Stated another way

$$G(s) = CPG \cdot G'(s) \qquad 3\text{-}10$$

The parameter KRL is thus defined as

$$KRL = K \cdot CPG/\tau$$

Substituting for A(s), H(s), and KRL and, rearranging 3-9 we are left with

$$\frac{X(s)}{X_c(s)} = \frac{KRL \cdot K_x}{s\left(\frac{1}{G'(s)} \right) + KRL\ (s^2 + K_x s + K_x)} \qquad 3\text{-}12$$

Equation 3-12 shows the effect of the SRFIMF gain parameter, KRL. As KRL increases, the model term $(s^2 + K_x s + K_x)$,

becomes more dominant in the closed loop transfer function. Using equation 3-12, we can examine the frequency response of the controller.

Assuming the plant transfer function to be a second order system given by G(s), equation 3-2, equation 3-12 can be rewritten as

$$\frac{X(s)}{X_c(s)} = \frac{KRL \cdot K_x}{s^3 + (KRL + b)s^2 + (KRL\ K_x + c)s + K_x \cdot KRL} \qquad 3\text{-}13$$

The dominant effect of KRL in equation 3-13 is again seen in the coefficients of the s and s^2 terms. From table 3-2 we see that a likely range in the values of b and c are $-0.6 \leq b \leq 1.6$ and $0.0 \leq c \leq 8.64$. If KRL is of the order of 25 and b and c are in the range of 2 and 8 respectively, then KRL will dominate the terms of equation 3-13 which contain b and c. Equation 3-13 can be simplified by neglecting b and c with resulting transfer function for the closed loop system written as

$$\frac{X(s)}{X_c(s)} = \frac{KRL \cdot K_x}{s^3 + KRL(s^2 + KRL\ K_x s + K_x)} \qquad 3\text{-}14$$

From this later representation it is clear that the plant dynamics which were determined by the coefficients b and c no longer play a role in the frequency response analysis. Equation 3-14 expressed in Bode form becomes

$$\frac{X(s)}{X_c(s)} = \frac{KRL \cdot K_x}{(i\omega)^3 + KRL(i\omega)^2 + KRL\ K_x(i\omega) + KRL\ K_x} \qquad 3\text{-}15$$

Equation 3-15 was plotted for frequencies from .01 to 100 and for KRL values of 1, 10, 25, 50 and 100. Figures 3-2 to 3-6 are the plots of the frequency response of the closed loop system. It can be seen that the frequency response of the system is that of a low pass filter with a break frequency of around 3.0 and a 40 db/Dec roll off. It is also seen that the response is very nearly in phase with the input for frequencies lower than 3.0. The results of the frequency response analysis show that the controller possesses good frequency response characteristics from the input to the output in that it has a flat response for all input frequencies of interest and little, if any phase shift. It has also been shown by Merrick [2] that the controller attenuates plant disturbances in the form of applied accelerations. The question of control disturbances will be examined from a different point of view in section V. We shall now consider the requirements for the gain, KRL, by root locus analysis.

C. ROOT LOCUS ANALYSIS

The intent of the root locus analysis is to determine the magnitude of the SRFIMF controller gain parameter, KRL, necessary for acceptable model following. Plots of the root locus of the oscillatory pole will show that the desired closed loop pole location is approached asymptotically as KRL is increased. Acceptable performance is determined by the specific application for which the SRFIMF controller is used. We begin by examining the equations for the closed loop system developed in the preceeding section.

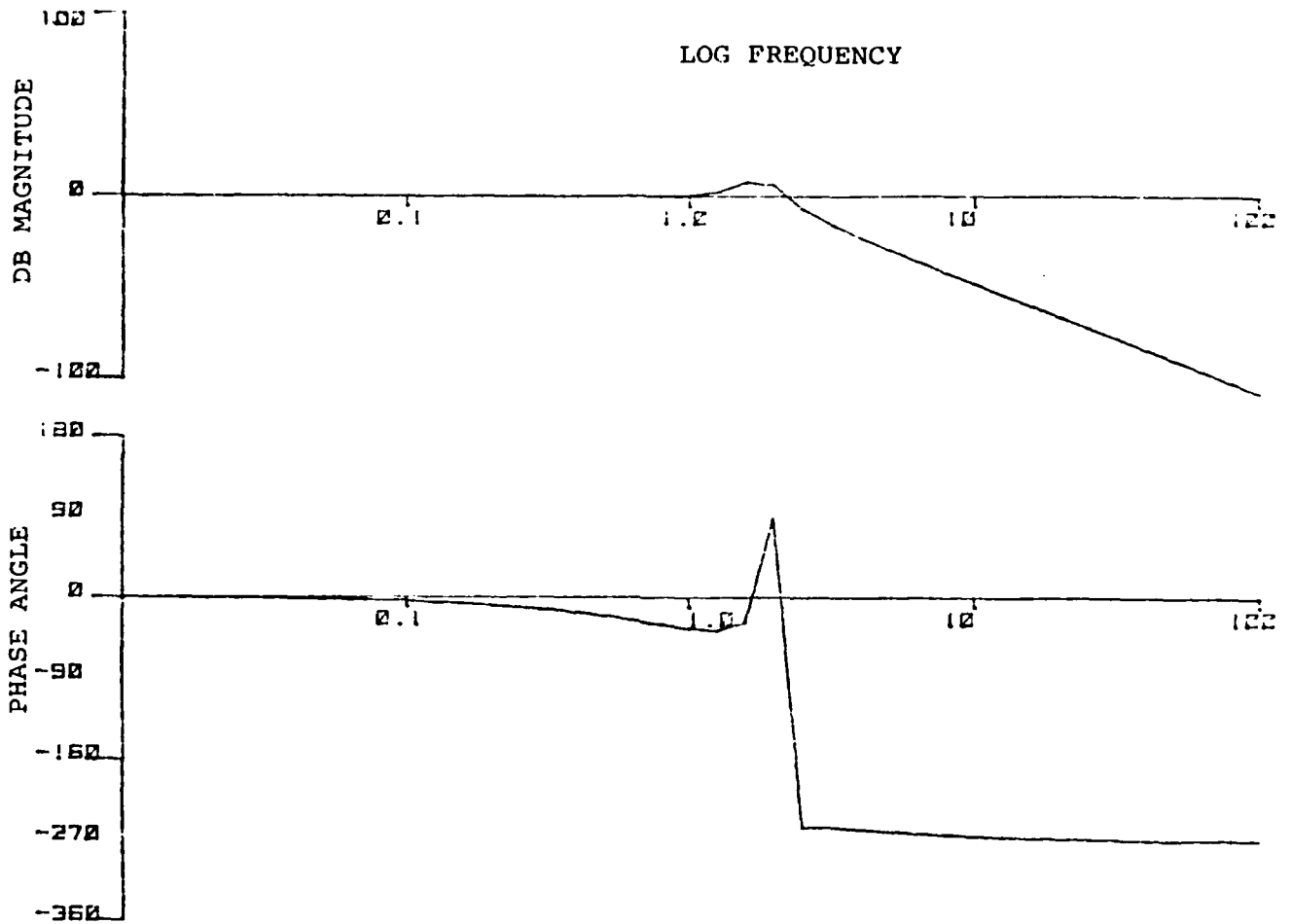

Figure 3-2 Frequency response of SRFIMF position controller, KRL = 1.

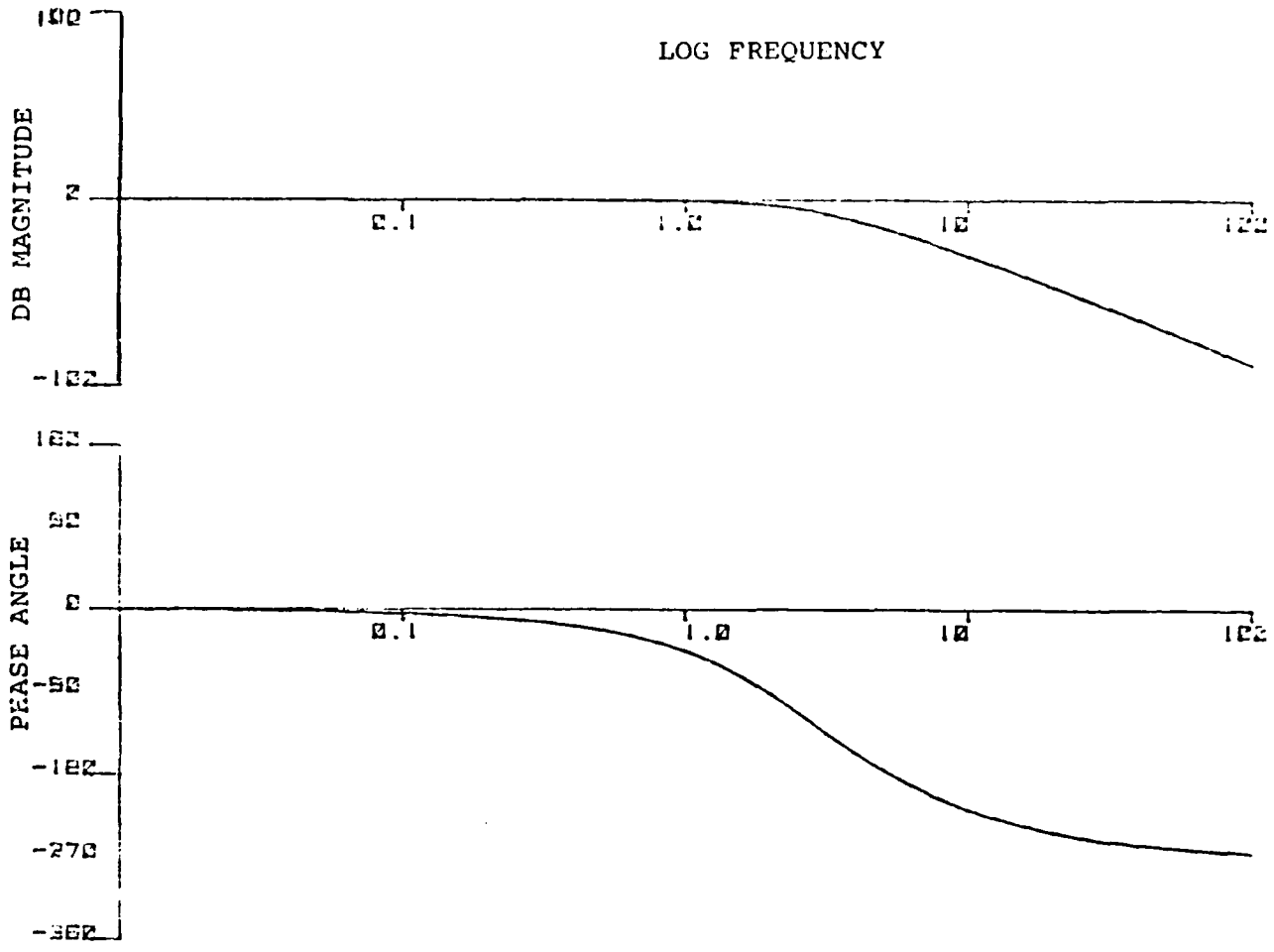

Figure 3-3 Frequency response of SRFIMF position controller, KRL = 10.

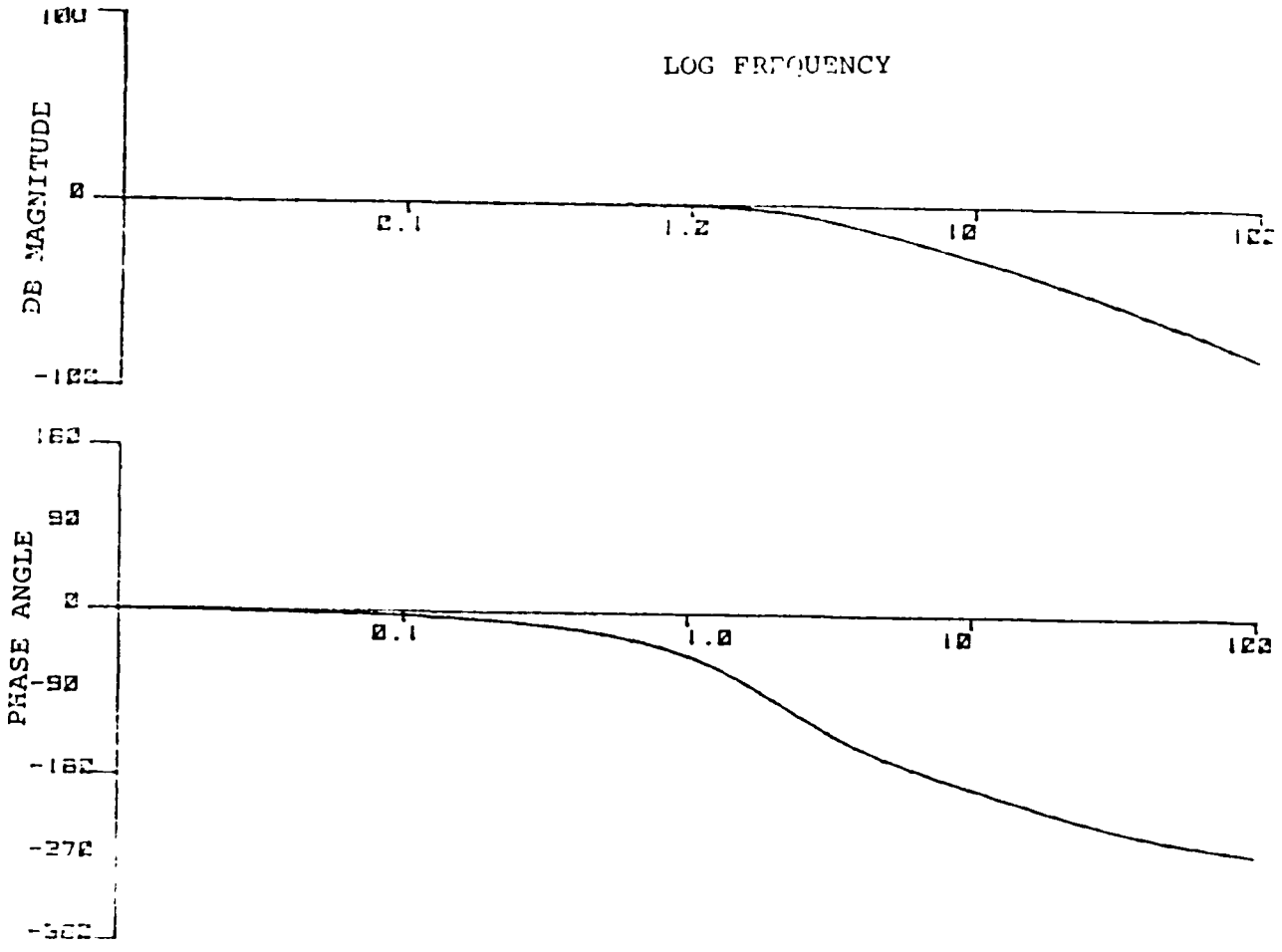

Figure 3-4 Frequency response of the SRFIMF position controller, KRL = 25.

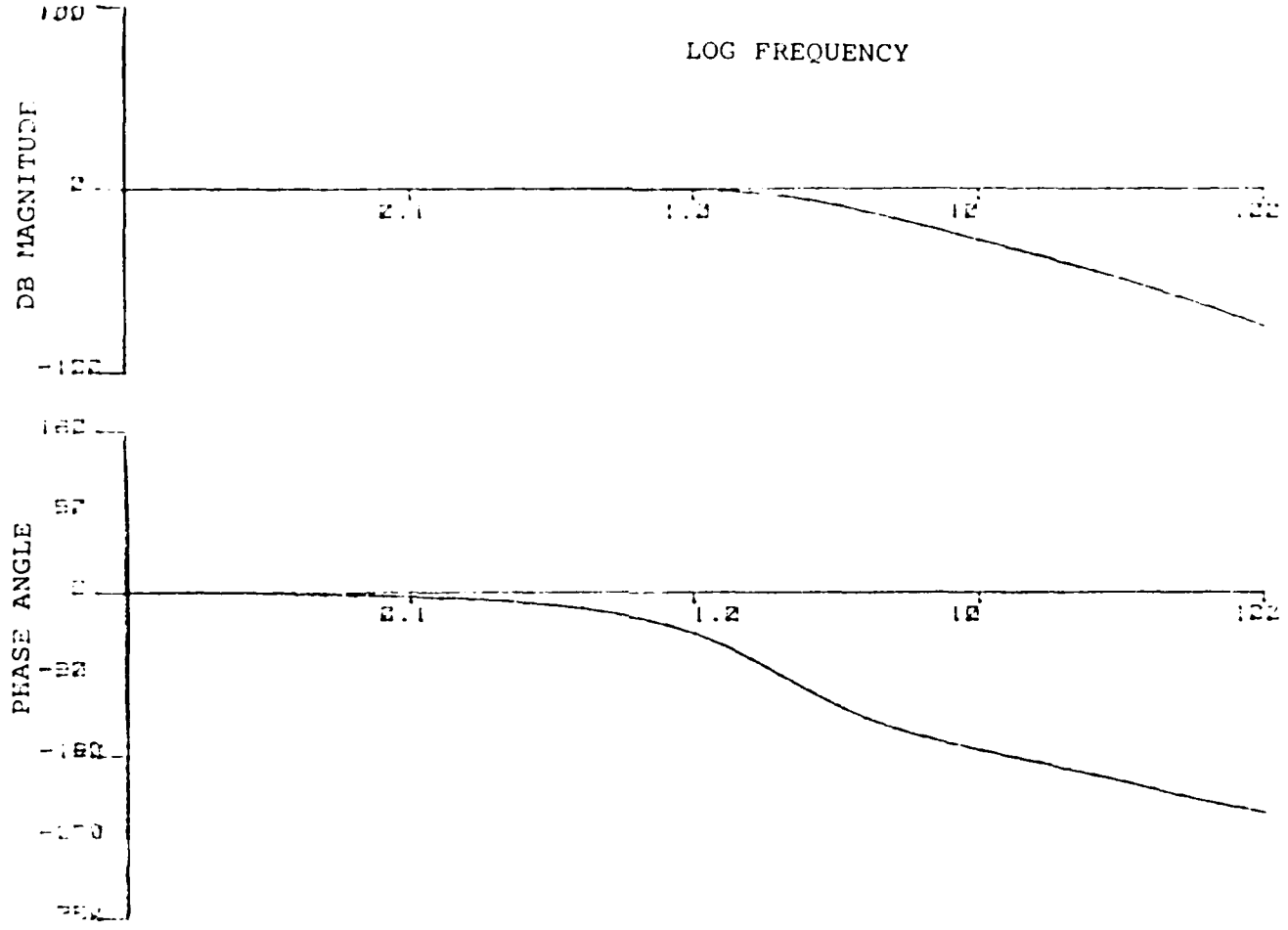

Figure 3-5 Frequency response of SRFIMF position controller, KRL = 50.

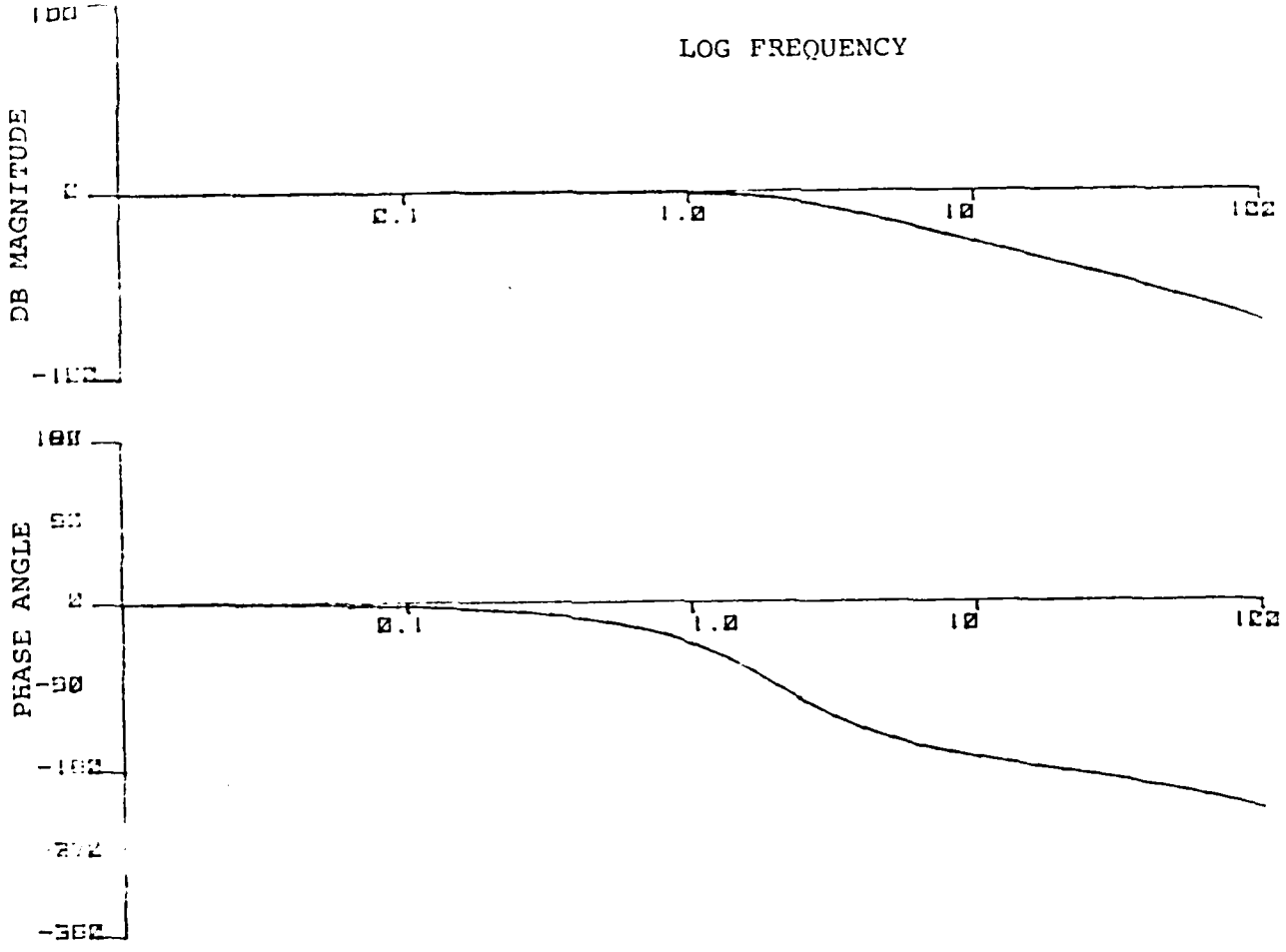

Figure 3-6 Frequency response of SRIMF position controller, KRL = 100.

The characteristic equation for the system can be obtained by setting the denominator of equation 3-13 to zero. The resulting equation is

$$s^3 + (b + KRL)s^3 + (c + K_{\dot{x}}\, KRL)s + KRL \cdot K_x = 0 \qquad 3\text{-}16$$

At least one root of 3-16 must be real and the factor corresponding to the real root is defined as (s + f). Assuming that the system described by the transfer function, equation 3-13, represents a model following system, then the second factor of 3-16 must be given by the model. In other words, assuming perfect model following, equation 3-16 must factor as

$$(s + f)\ (s^2 + K_{\dot{x}}s + K_x) = 0 \qquad 3\text{-}17$$

As KRL becomes large, the SRFIMF closed loop system response does approach perfect model following, and it can be stated that as KRL increases the condition is approached where

$$s^3 + (b + KRL)s^2 + (c + K_{\dot{x}}KRL)s + KRL \cdot K_x \longrightarrow$$
$$(s + f)\cdot(s^2 + K_{\dot{x}}s + K_x) \qquad 3\text{-}18$$

In other words, the dynamic behavior of the system's oscillatory mode is approximately given by the model parameters $K_{\dot{x}}$ and K_x. Expanding the right hand side of 3-18 we have for increasing KRL

$$s^3 + (b + KRL)s^2 + (c + K_{\dot{x}}KRL)s + KRL \cdot K_x \longrightarrow$$
$$s^3 + (K_{\dot{x}} + f)s^2 + (K_x + K_{\dot{x}}f)s + f \cdot K_x \qquad 3\text{-}19$$

Comparing coefficients reveals that for increasing KRL

$$b + KRL \rightarrow K_x + f \qquad 3\text{-}20a$$

$$c + K_x KRL \rightarrow K_x f + K_x \qquad 3\text{-}20b$$

$$KRL \rightarrow f \qquad 3\text{-}20c$$

The relationships of 3-20 are not equalities, but they indicate that while KRL is increasing, the difference between KRL and f remains finite.

Therefore, it can be said that one real pole is on the negative real axis and its location is approximated by the value of KRL since f is approaching KRL as KRL becomes large relative to b or c. To examine the behavior of the oscillatory pole as KRL is increased, it will be necessary to rearrange equation 3-9 into root locus form. Setting the denominator of 3-9 equal to zero yields

$$\left[\frac{1 - A(s)}{K}\right] \cdot \left[\frac{1}{H(s)\ G(s)}\right] + s^2 + K_x s + K_x = 0 \qquad 3\text{-}21$$

Substituting for A(s), H(s) and G(s) and rearranging, the general relation for the rool locus is

$$0 = 1 + \frac{KRL\ (s^2 + K_x s + K_x)}{s}\ G'(s) \qquad 3\text{-}22$$

where G'(s) is an arbitrary plant transfer function divided by its constant gain, CPG, as was shown by equation 3-10.

Equation 3-21 will now be used to determine the root locus for the position controller where G'(s) is given by one of four of the modes assumed to be typical of VSTOL aircraft as listed in table 3-4. The cases considered are

	Mode # from Table 3-2	b	c
Case 1	6	-.6	2.16
Case 2	3	.6	.91
Case 3	9	0	0
Case 4	10	1.6	.6

Table 3-4 Plant Parameters for Root Locus Analysis

The root locus computer program developed by Melsa and Jones [7] was used to evaluate and plot the root locus given by equation 3-21. The gain constant, KRL, was varied from 0 to 100. The resulting root locus trajectory of the oscillatory poles for the four mode cases are shown in figures 3-7 to 3-10. From the figures it can be seen that in all cases the oscillatory pole approaches the desired value given by the model, in this case -1.5, 1.32i. Because of scaling, the real pole described earlier is not shown in the figures. It can also be seen that the pole location of the closed loop system is within 5 percent of the desired value for KRL of between 25 and 50. Although a value of KRL equal to 25 is normally sufficient, a KRL value of 50 will be used for the remainder of this work.

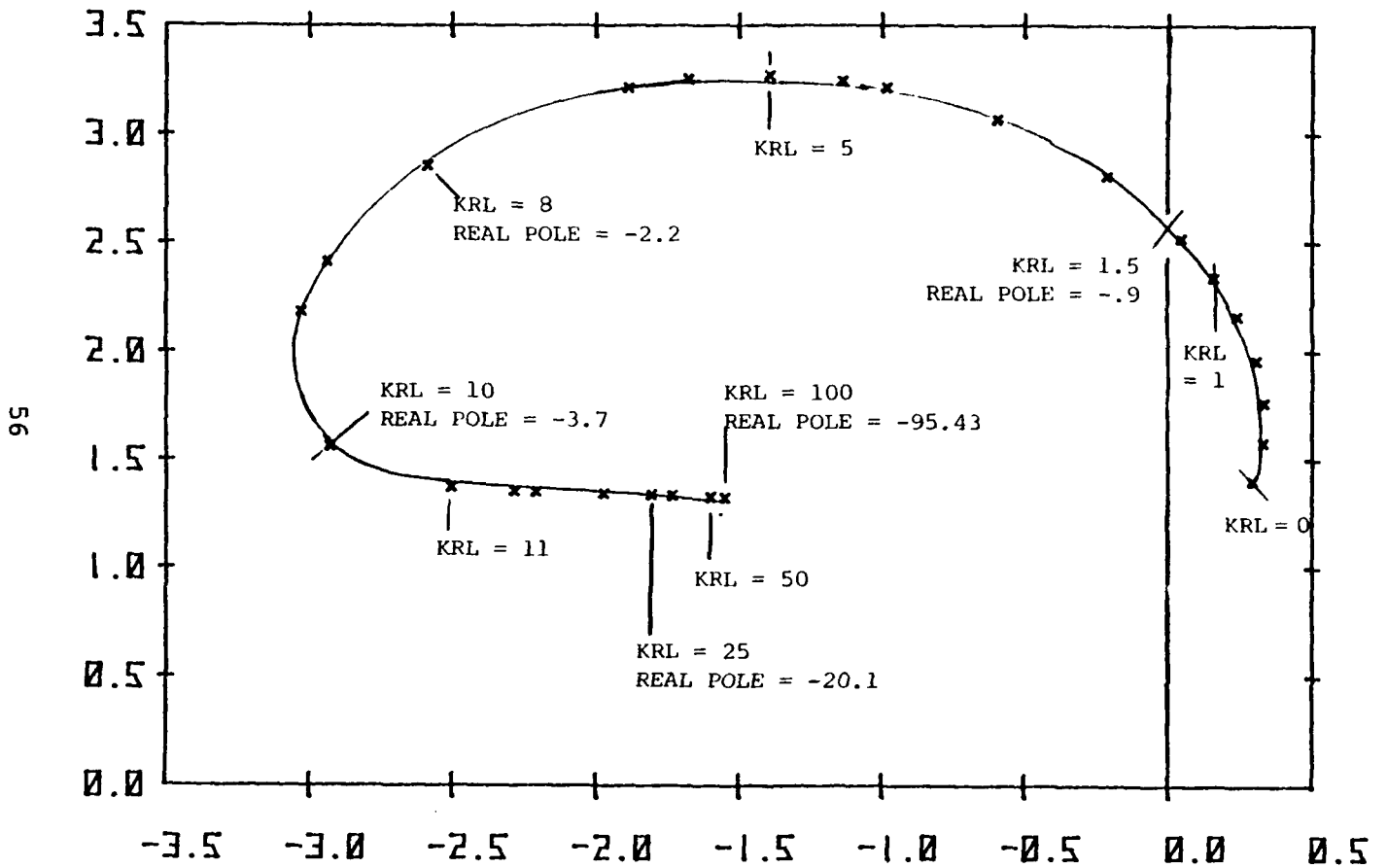

Figure 3-7 Root locus plot of oscillatory pole of a SRFIMF position controller. The open loop plant has a natural frequency of 1.47 rad/sec and a damping ratio of -.441.

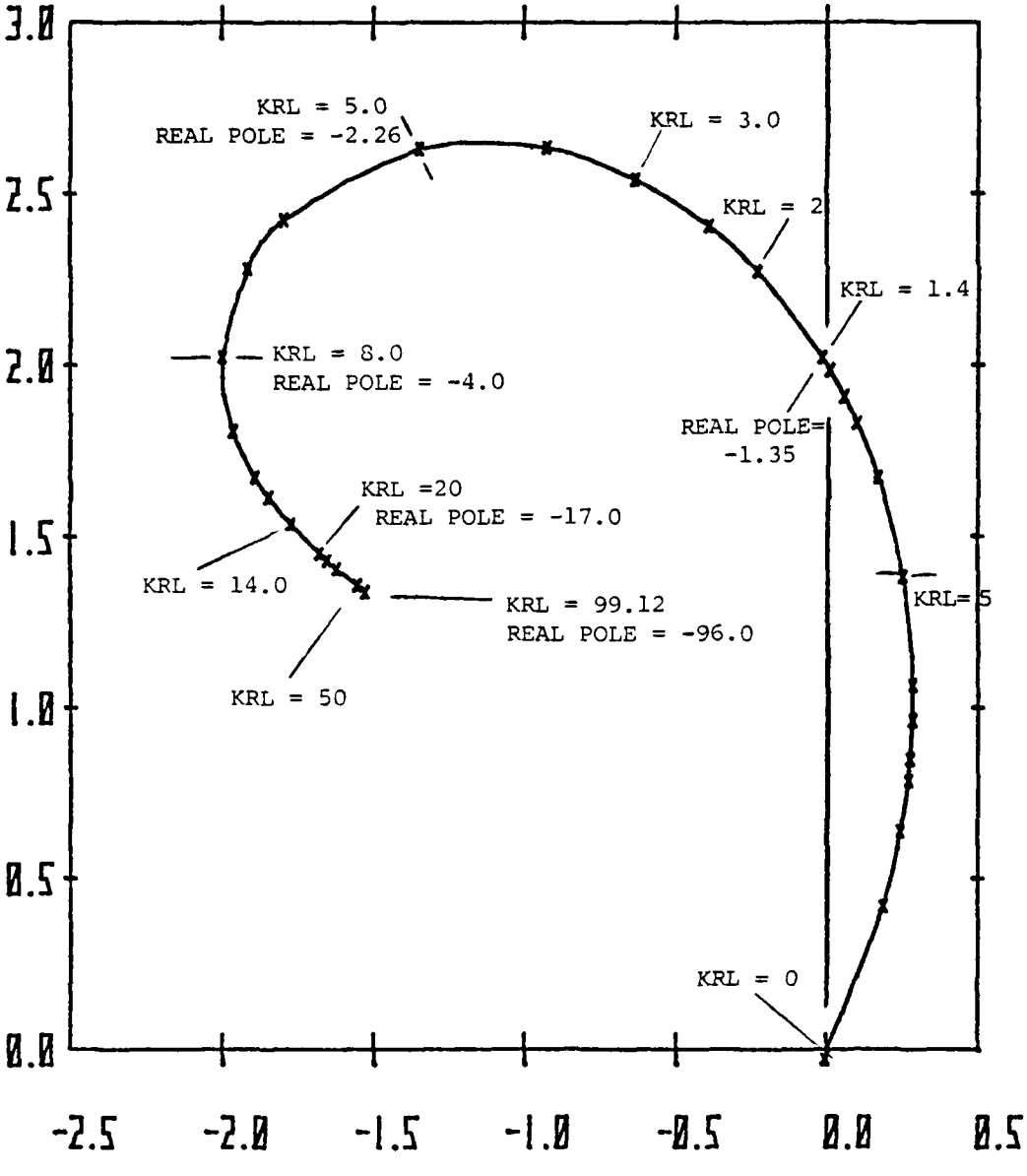

Figure 3-8 Root locus of the oscillatory pole of a SRFIMF position controller. The open loop plant has two poles at the origin.

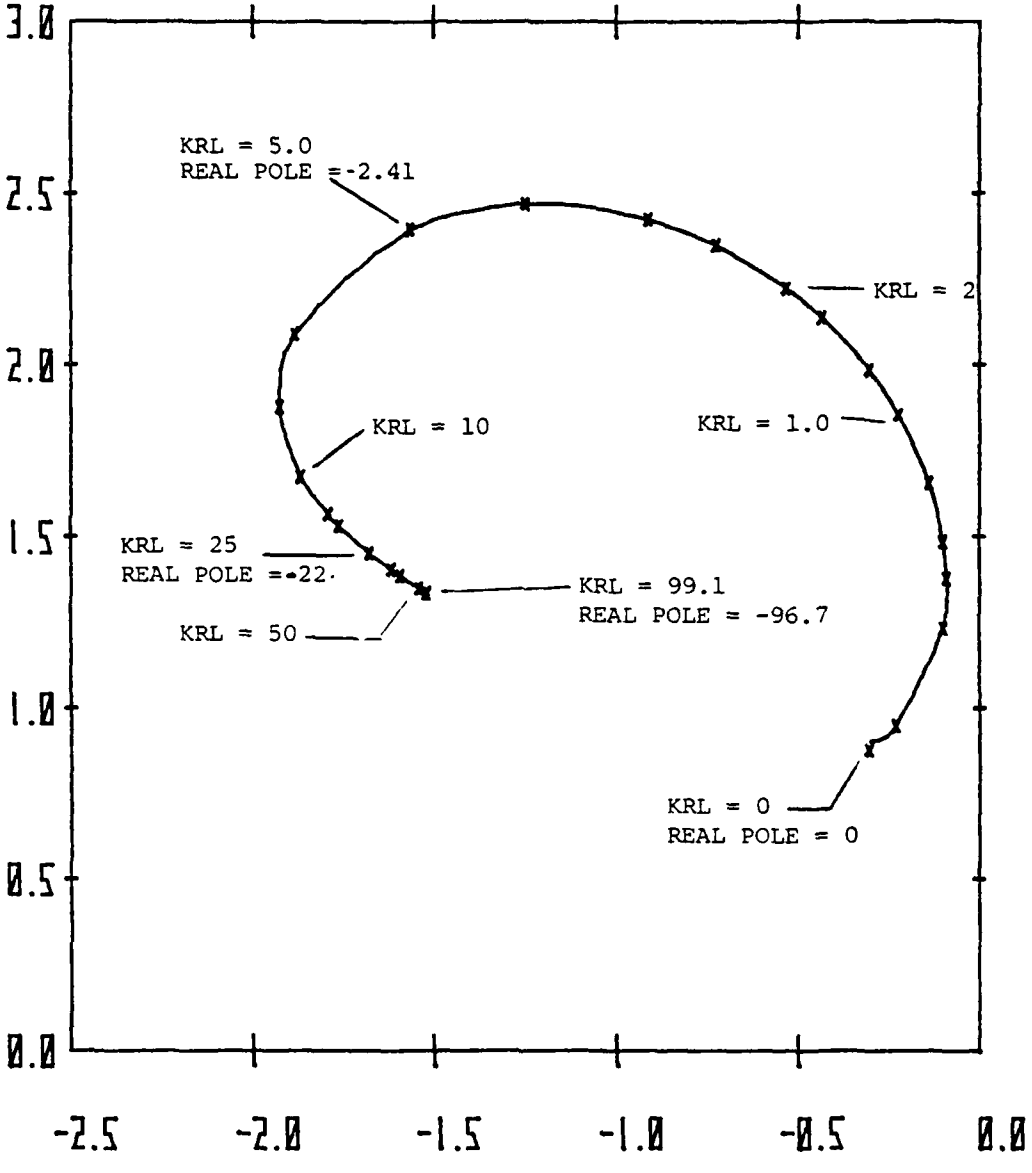

Figure 3-9 Root locus of the oscillatory pole of a SRFIMF controller. The open loop plant has a natural frequency of .95 rad/sec and damping ratio of 0.475.

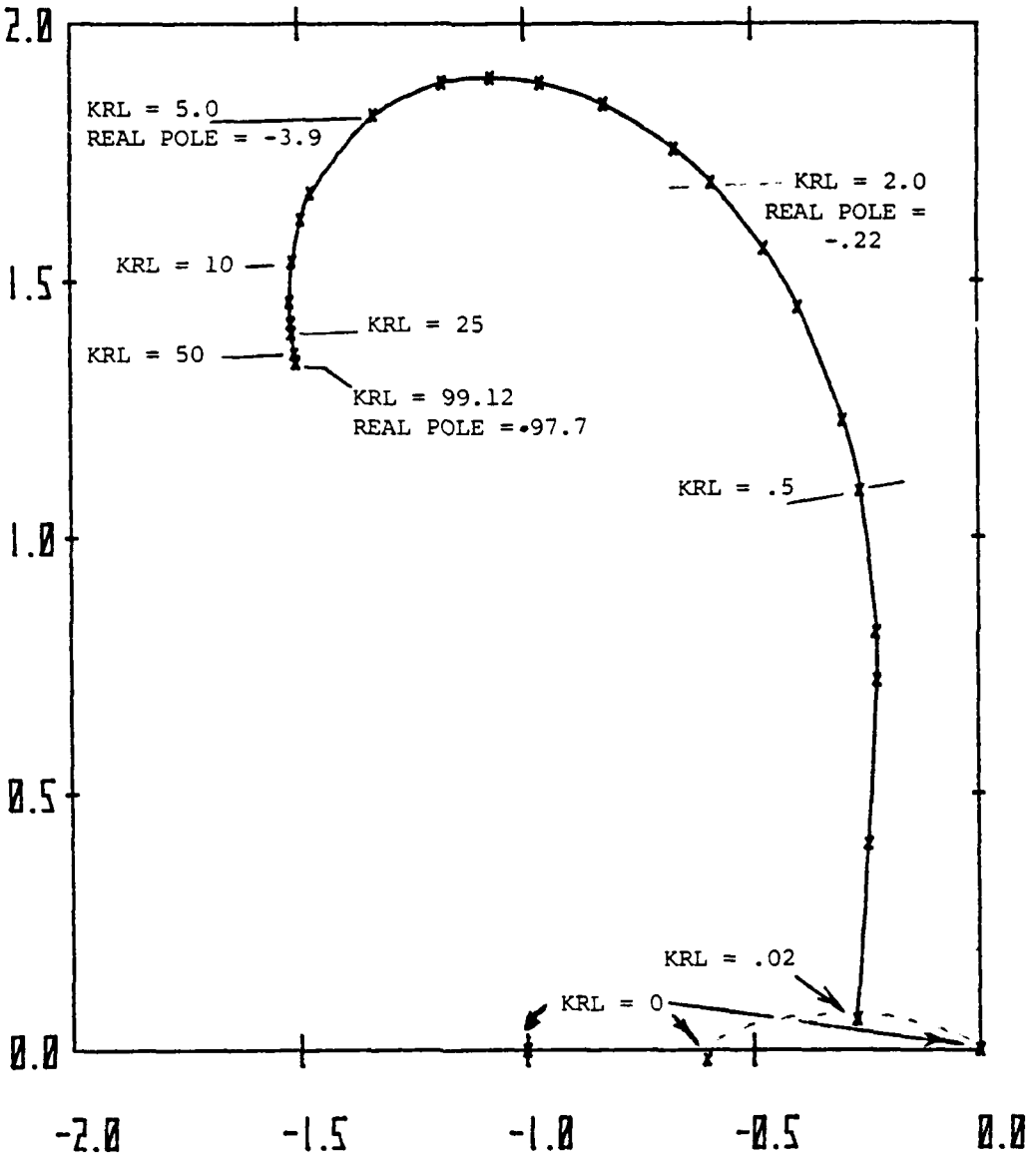

Figure 3-10 Root locus of the oscillatory pole of a SRFIMF position controller. The open loop plant has real poles at -1.0 and -.6.

The root locus behavior of a more complex plant will be shown in section V where the SRFIMF controller will be applied to a transfer function of the Harrier aircraft. The system will next be simulated for each of the sample VSTOL aircraft modes listed in table 3-2.

D. SRFIMF SIMULATIONS

The simulated response of a plant under the control of SRFIMF position controller was done for all ten of the typical VSTOL aircraft modes listed in table 3-2. The time histories were generated using the CSMP program, reference [6]. The open loop plant for each of the ten systems was assumed to be of the form

$$G(s) = \frac{X(s)}{\delta(s)} = \frac{CPG}{s^2 + bs + c}$$

with CPG = 1 and b and c given in table 3-2.

The model was as assumed in section II, a second order with a natural frequency of 2 rad/sec and damping ratio, of 0.75. The constants $K_{\dot{x}}$ and K_x were 3 and 4 respectively as determined by equations 2-8 and 2-9. The simulation of the model response is shown in figure 3-9. The rise time of the modeled response is approximately 1.2 seconds. The model response has a slight overshot (2.9 percent) with a peak time at 2.4 seconds. Settling to within 1 percent occurs immediately after the peak response. For the simulations, KRL was taken to be 50 and the actuator time constant, τ, was 0.10 seconds. The

input to the system and to the open loop plant was a unit step function, representing X_c, at t = 0. A sample listing of the CSMP source code is given in Appendix A.

Figures 3-11 to 3-21 show the time histories of the open and closed loop response of each of the ten simulated systems. It can be seen from the figures that the closed loop response of each system is indistinguisable from the model response. Figures 3-11 to 3-21 graphically show that the response of the closed loop system is approximated by the model response regardless of the plant being controlled. In addition, the self trimming feature of the controller is seen. In all cases the steady state value of the output of the closed loop system is the same as the input, X_c, namely 1.

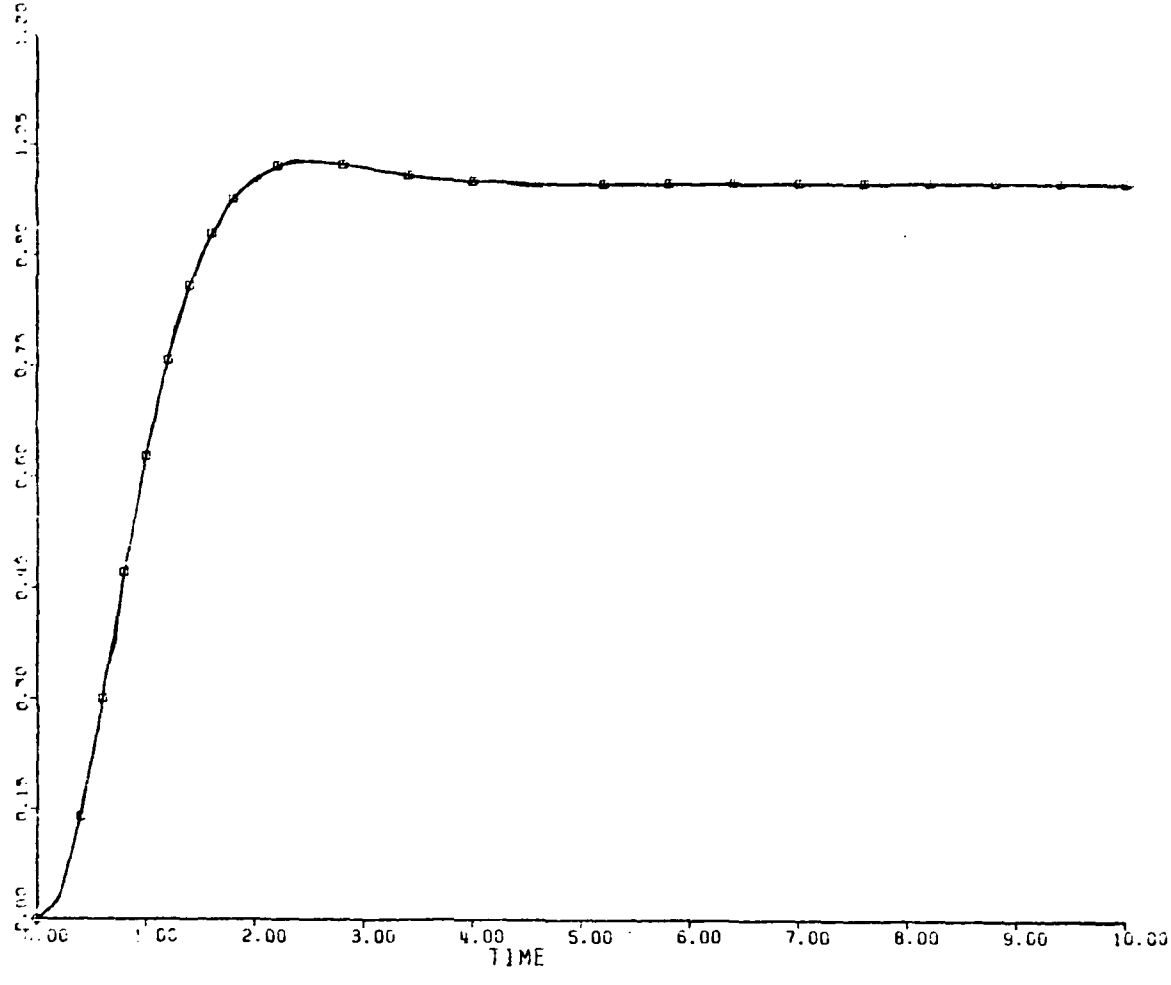

Figure 3-11 Perfect model response. $K_x = 4$, $K_{\dot{x}} = 3$, $\omega_n = 2$, $\zeta = 0.75$.

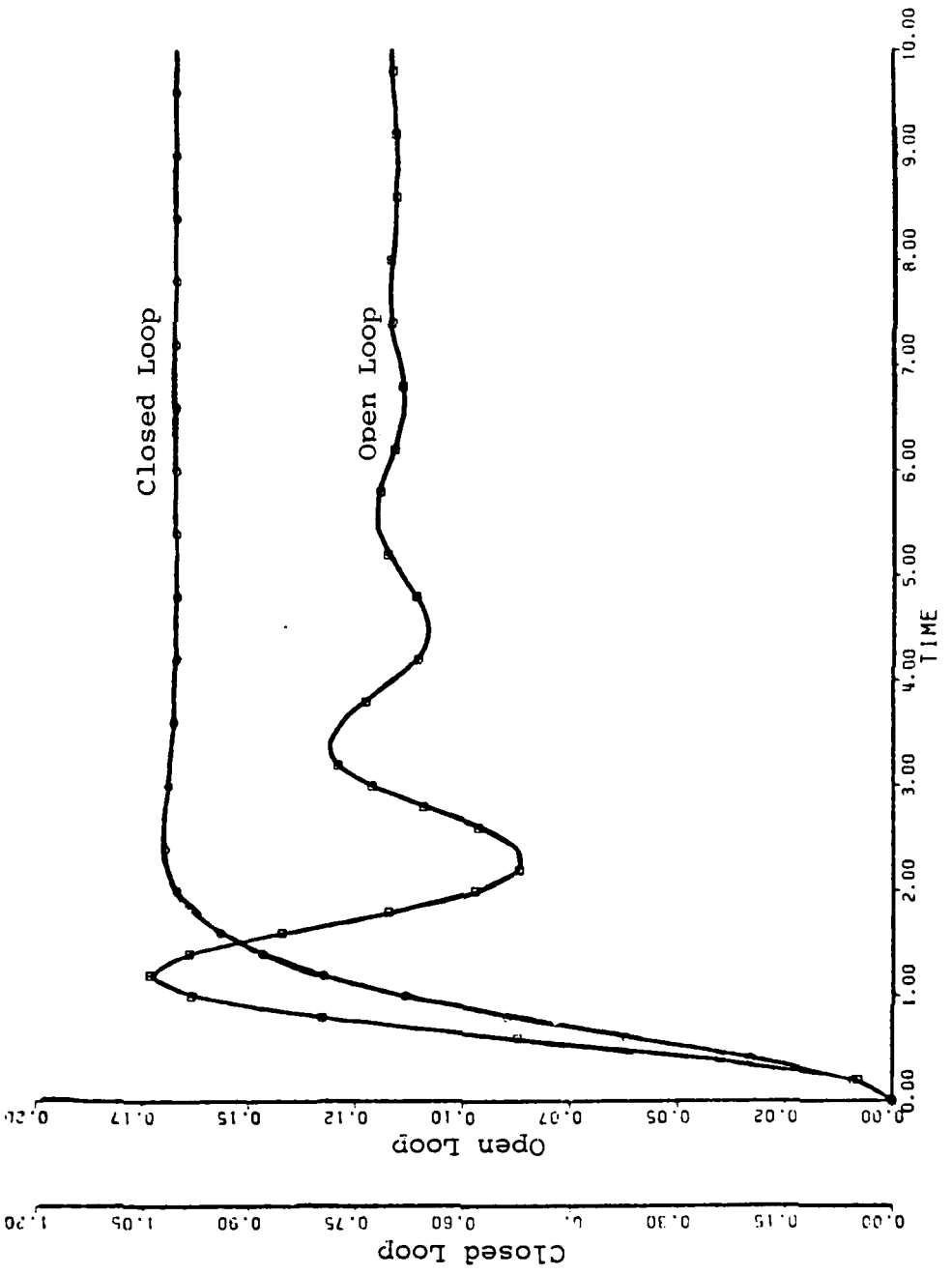

Figure 3-12 Simulation of SRFIMF position control for aircraft mode 1, table 3-3. The open loop plant has a natural frequency of 2.94 rad/sec and damping of .2.

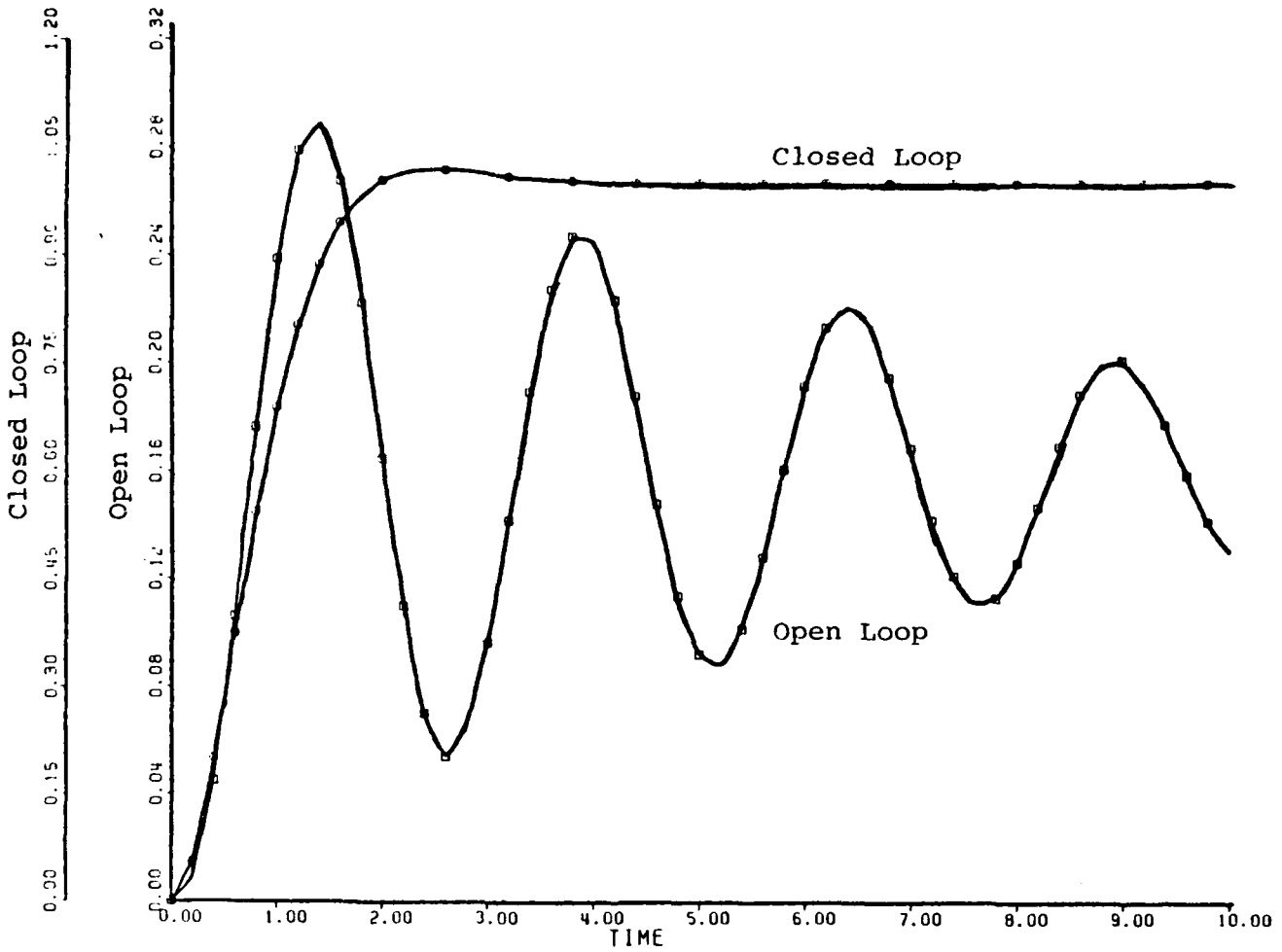

Figure 3-13 Simulation of SRFIMF position control for aircraft mode 2, table 3-3. The open loop plant has a natural frequency of 2.5 rad/sec and damping ratio of 0.06.

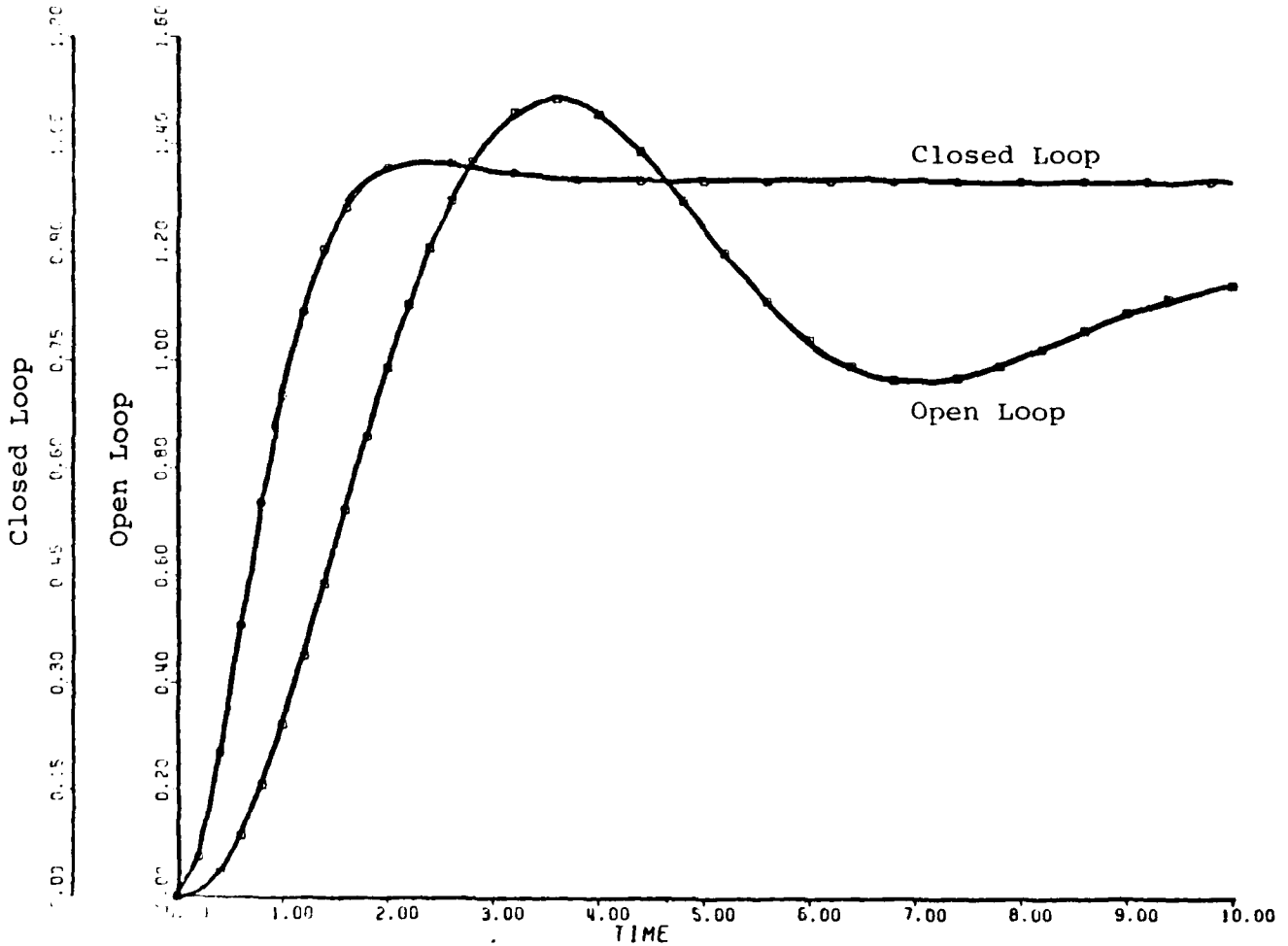

Figure 3-14 Simulation of SRFIMF position control for aircraft mode 3, table 3-3. The open loop plant has a natural frequency of 0.954 rad/sec and a damping ratio of 0.314.

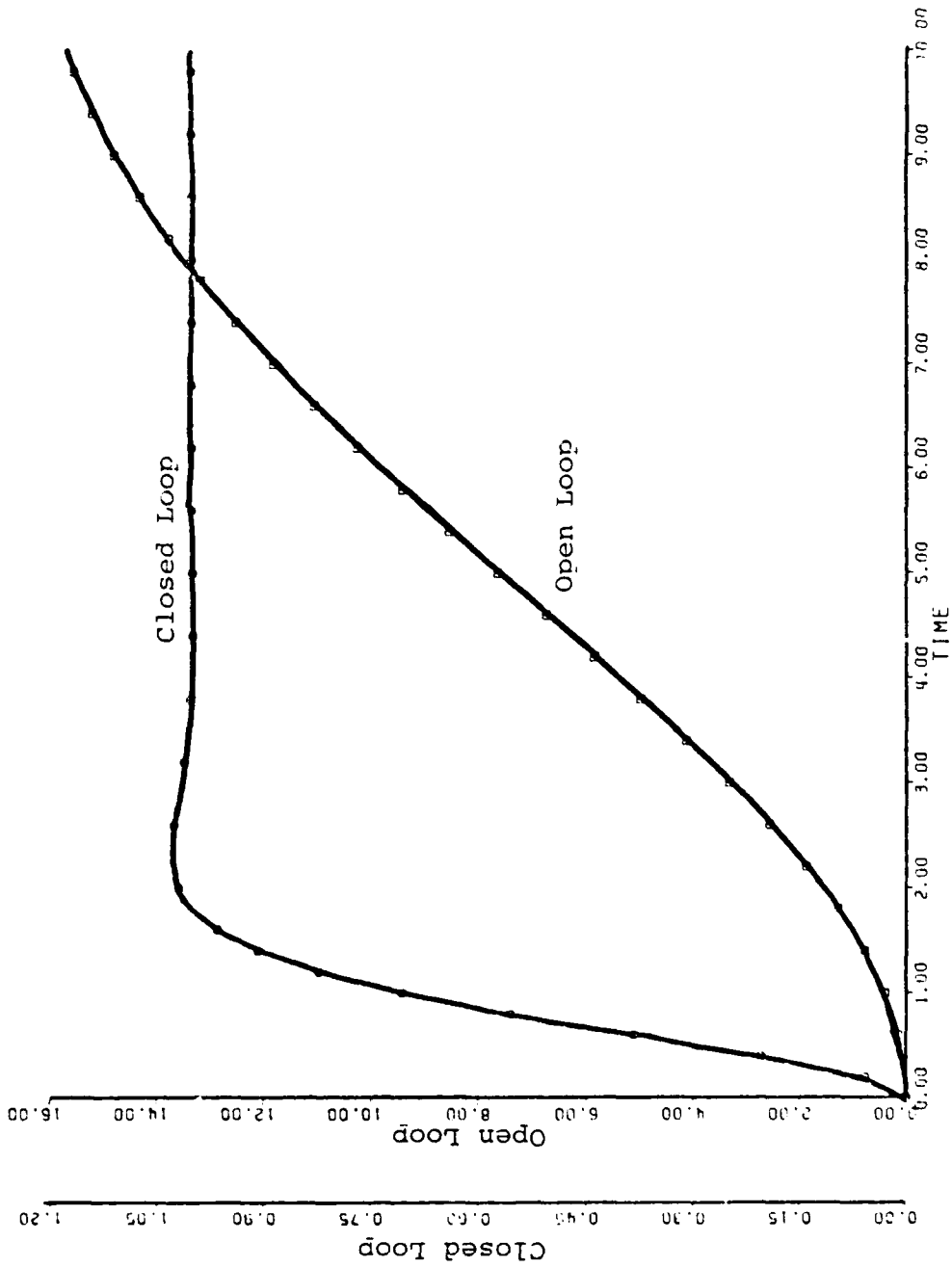

Figure 3-15 Simulation of SRFIMF position control for aircraft mode 4, table 3-3. The open loop plant has a natural frequency of 0.283 rad/sec and damping ratio of 0.353.

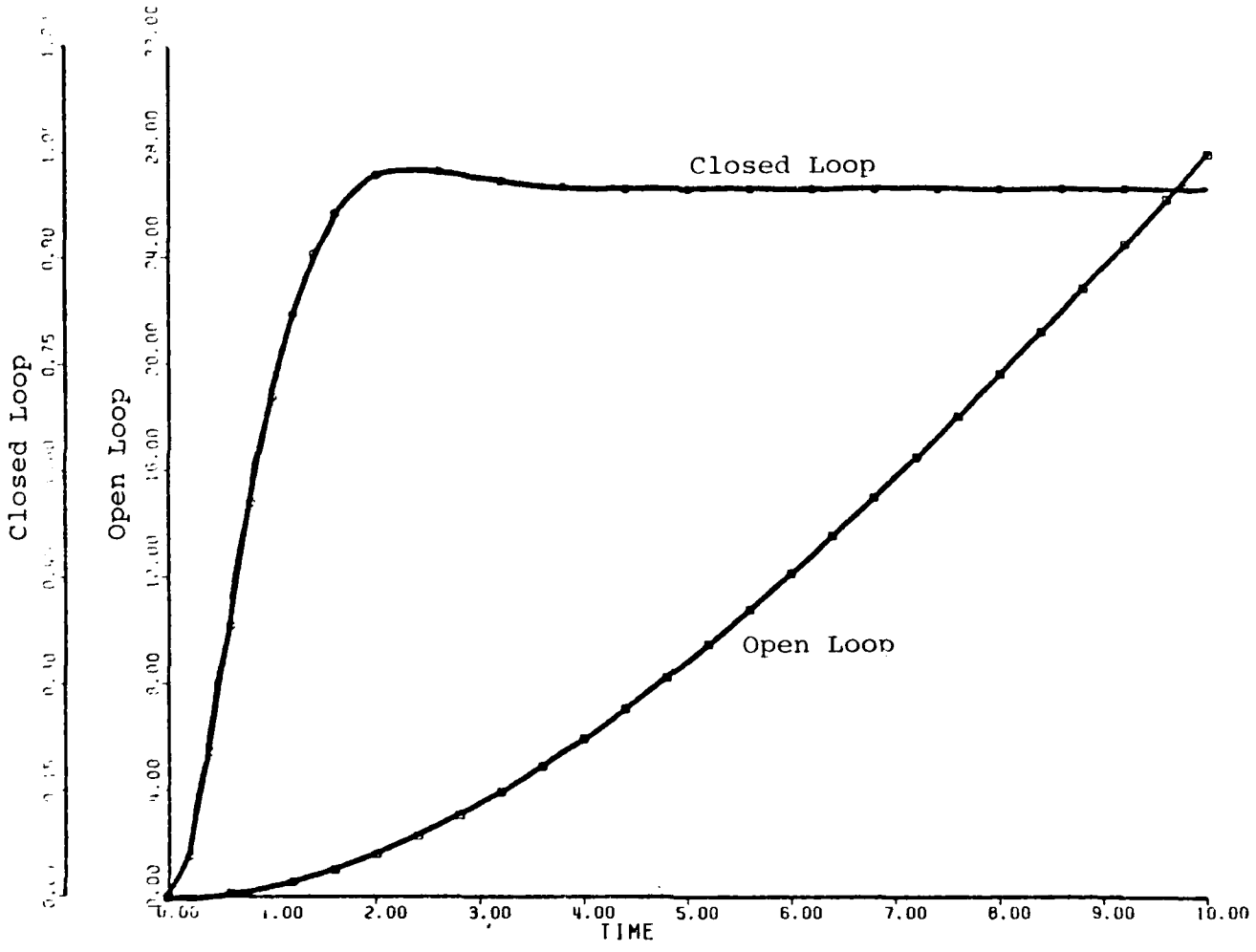

Figure 3-16 Simulation of SRFIMF position control for aircraft mode 5, table 3-3. The open loop plant has real poles at -0.2 and 0.

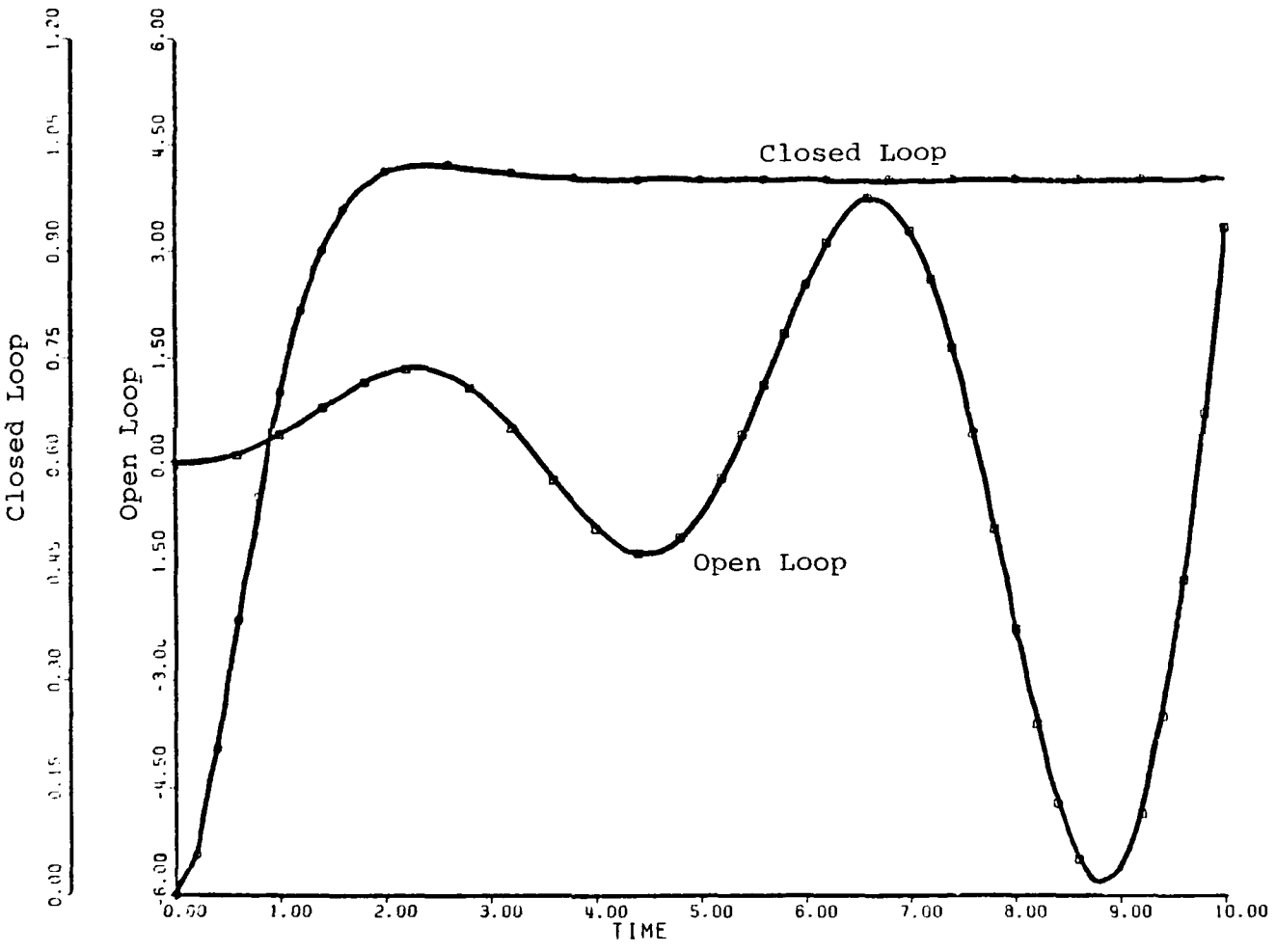

Figure 3-17 Simulation of SRFIMF position control for aircraft mode 6, table 3-3. The open loop plant has a natural frequency of 1.47 rad/sec and damping ratio of -.204.

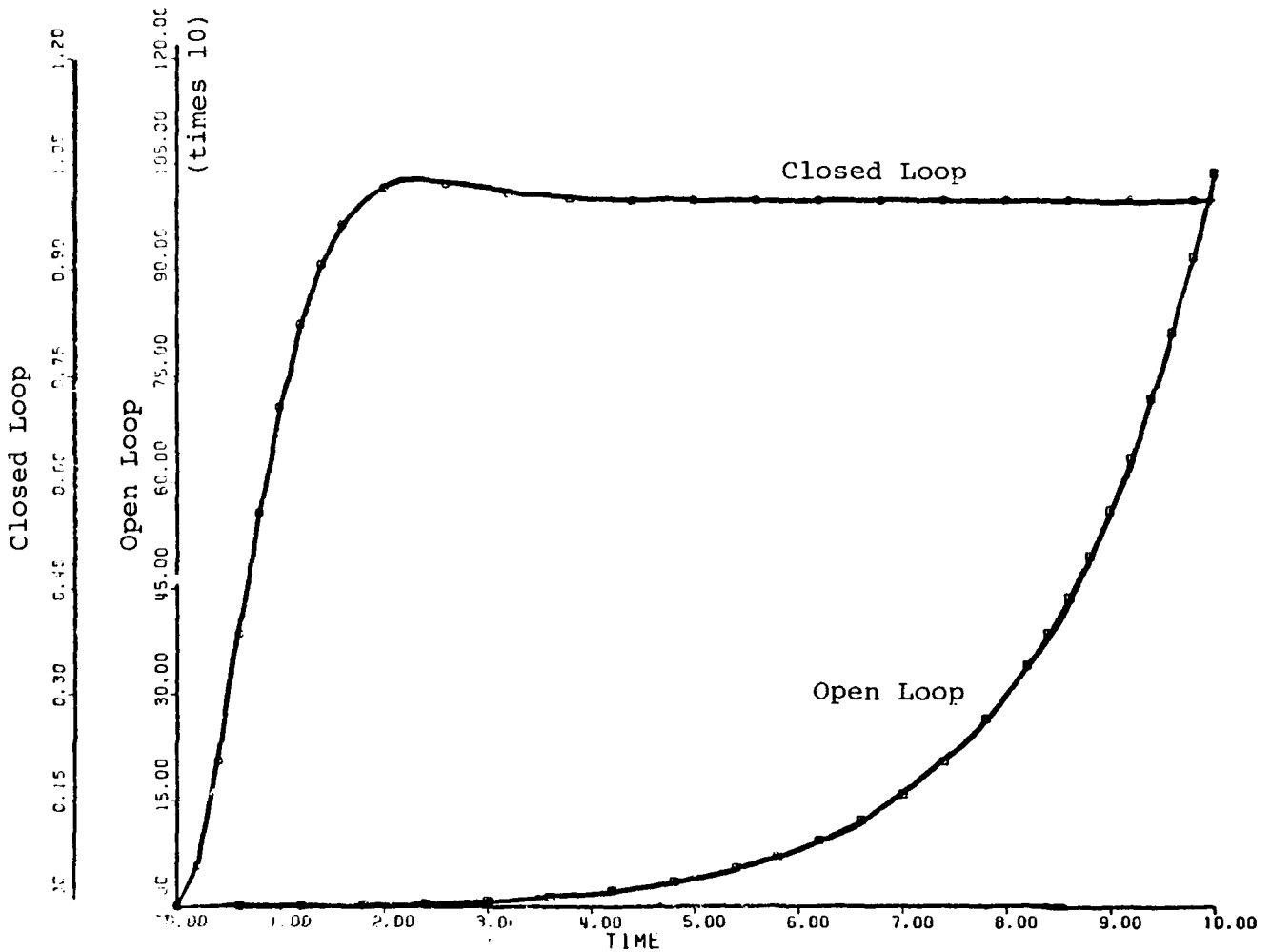

Figure 3-18 Simulation of SRFIMF position control for aircraft mode 7, table 3-3. The open loop plant has real poles at 0.30 and 0.

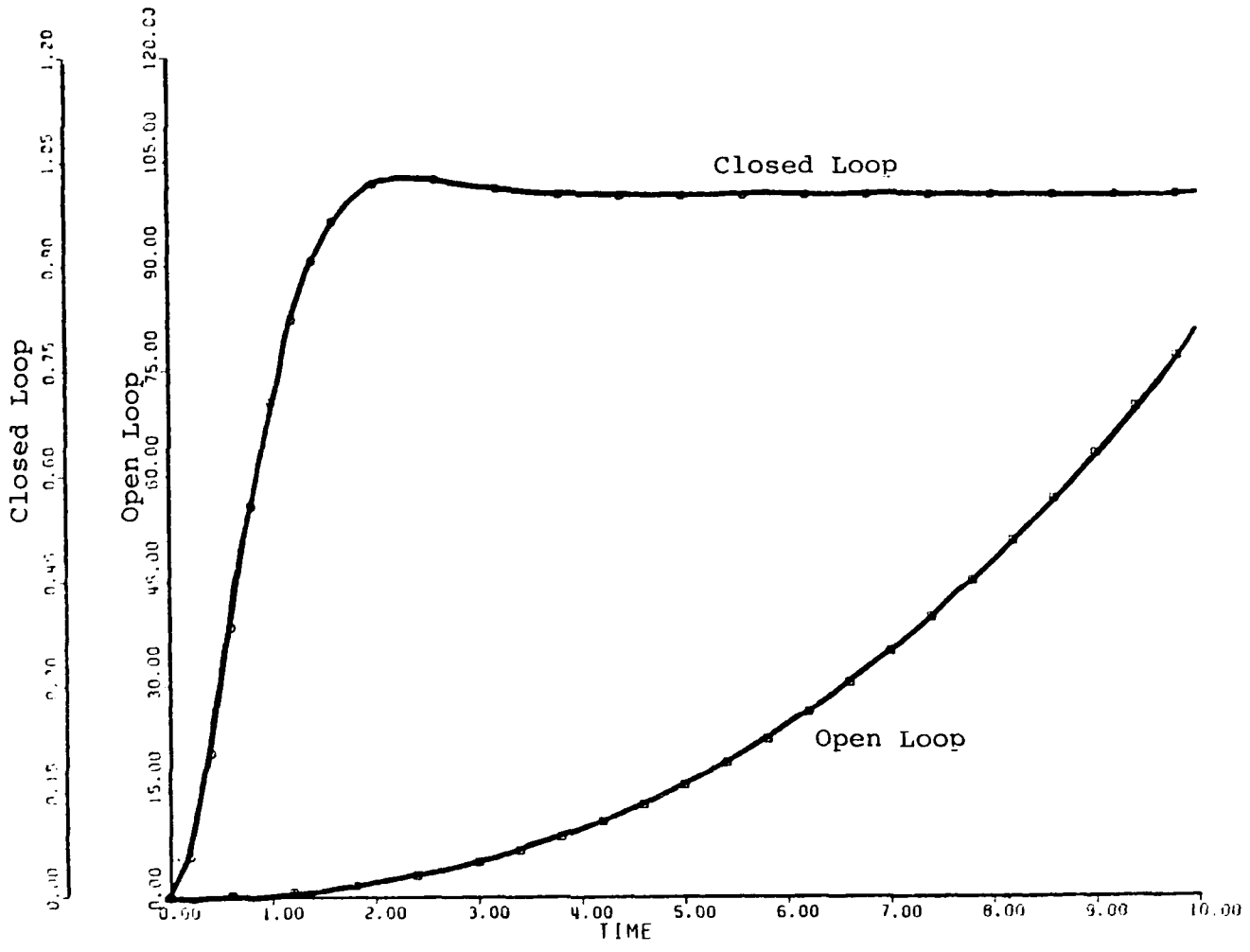

Figure 3-19 Simulation of SRFIMF position control for aircraft mode 8, table 3-3. The open loop plant has a natural frequency of 0.17 rad/sec and damping ratio of 0.590.

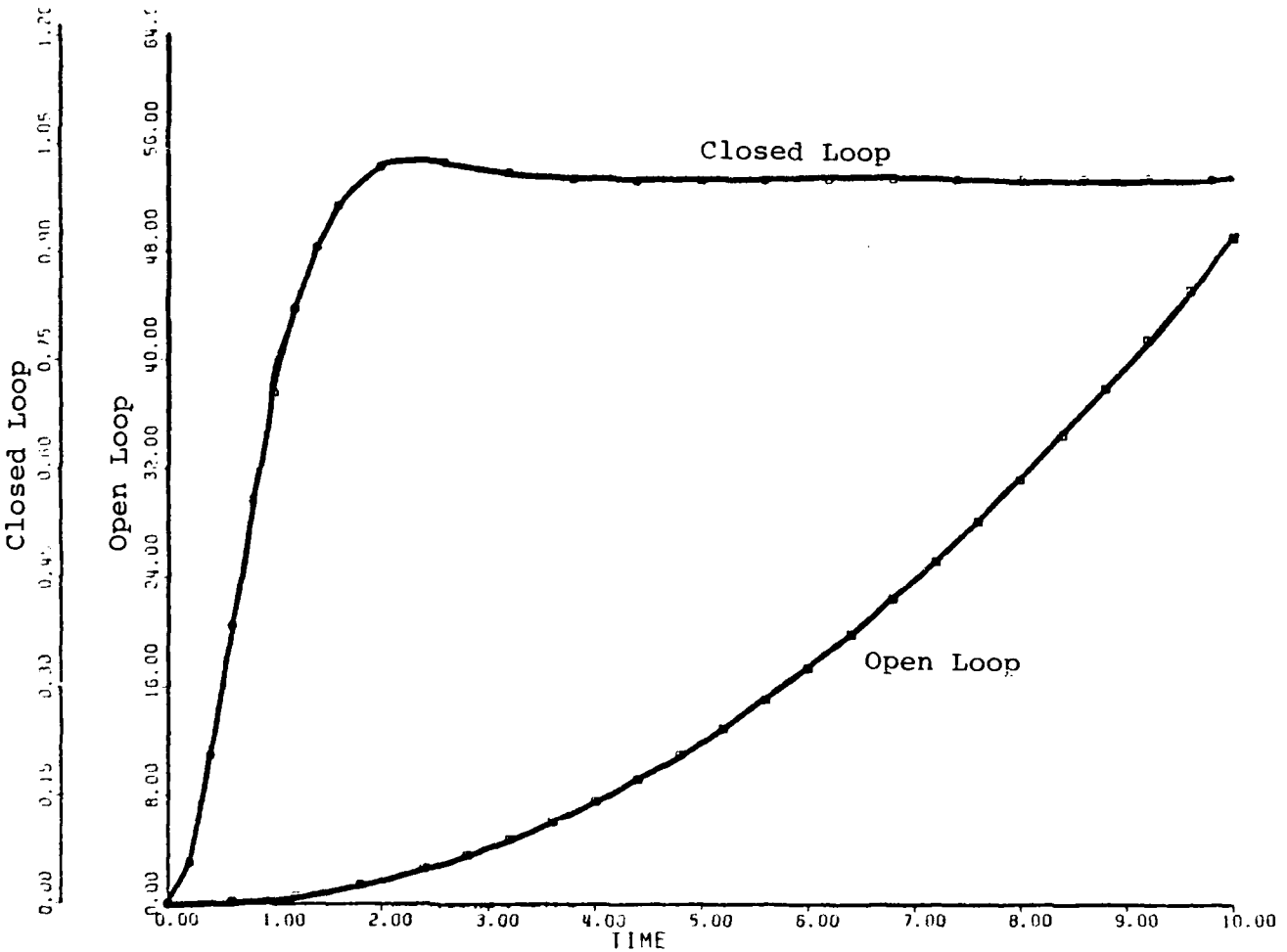

Figure 3-20 Simulation of SRFIMF position control of aircraft mode 9, table 3-3. The open loop plant has two poles at the origin.

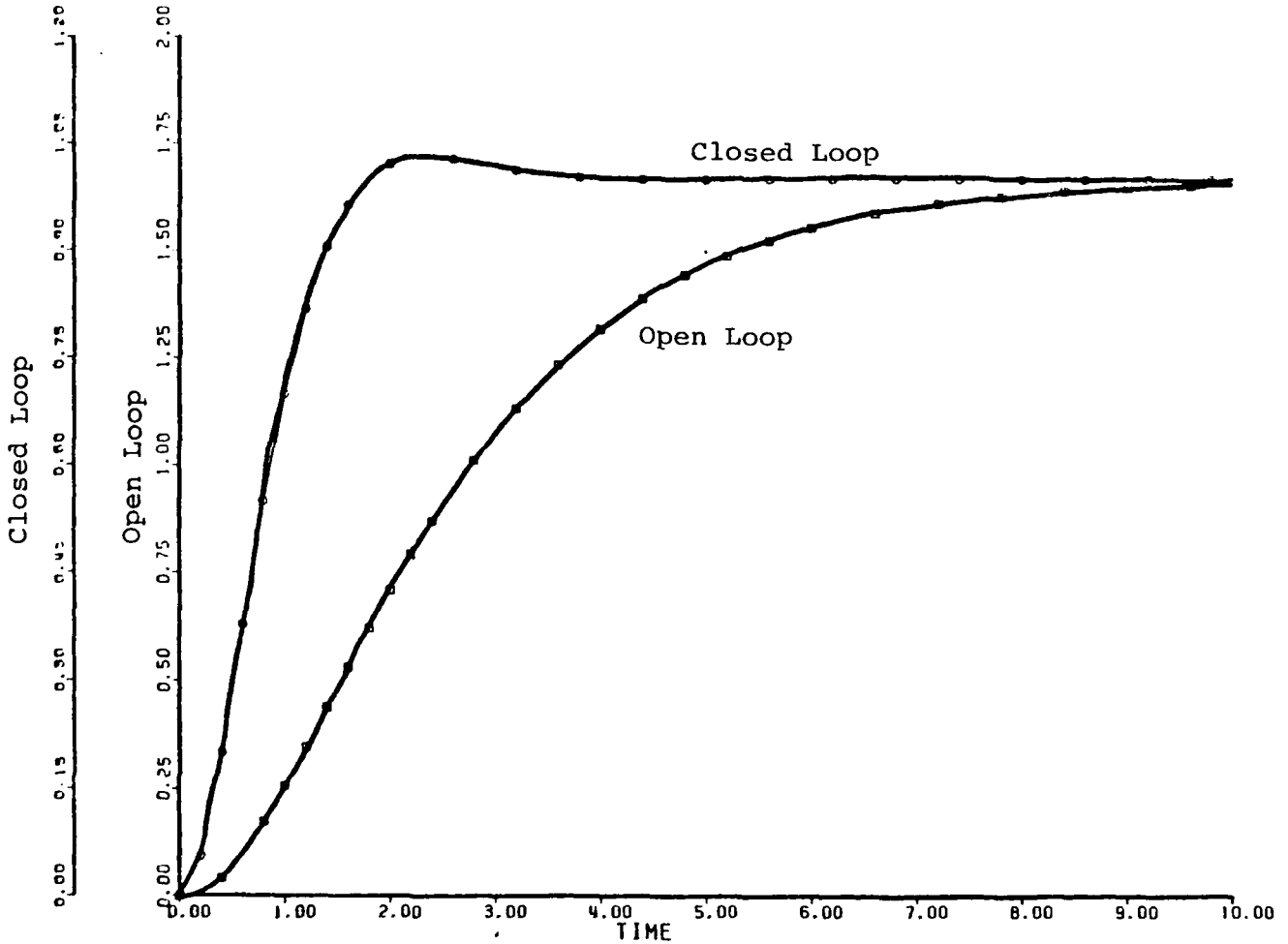

Figure 3-21 Simulation of SRFIMF position control of aircraft mode 10, table 3-3. The open loop plant has real poles at -1.0 and -0.6.

IV. SENSOR NOISE ANALYSIS

A. SRFIMF STOCHASTIC MODEL

The SRFIMF controller relies upon measurements to produce model following. We shall now examine the effect of measurement uncertainty on the operation of the closed loop system. In order to accomplish this we will consider a position controller with a representative second order plant given by

$$G(s) = \frac{CPG}{s^2 + bs + c} \qquad 4\text{-}1$$

The position controller will be augmented with sensor noise sources and the observability and controllability of the closed loop system with these noise sources will be analyzed. Errors in the state variables will be determined by covariance analysis using the Lyapunov equation.

The measured quantities that are considered to be contaminated by sensor noise are the attitude position, attitude rate and attitude acceleration measurements. Two types of sensors will be considered. One is of high accuracy and typical of good quality inertial navigation system measurements. The second is of lower accuracy and might be considered typical of strapdown sensors. Sensor errors are assumed to be of two types, bias and high frequency. Bias errors are usually small relative to measured quantities and they are of constant value. Bias errors have little effect on the dynamic behavior of the control system and will not be considered in this study.

High frequency errors can be modeled in various ways. It is assumed that the sensor noise sources can be modeled by exponentially correlated noise. Exponentially correlated noise is obtained by passing Gaussian white noise through a first order shaping filter. A first order filter is represented by the differential equation

$$\dot{\xi} = -1/T_r \xi + W_t(t) \qquad 4\text{-}2$$

where $W_t(t)$ is a scalar white, zero mean, Gaussian noise of constant strength, μ. μ is chosen so that the steady state value produced by the filter is the square of the sensor error standard deviation, σ^2. Following Maybeck [9], the required value of μ as the input to the filter is given by

$$\mu = 2\sigma^2/T_r \qquad 4\text{-}3$$

where T_r is the correlation time of the exponentially correlated noise. Figure 4-1 is a schematic representation of a typical shaping filter, in Laplace transform notation. $\xi(s)$ is resulting exponentially correlated noise.

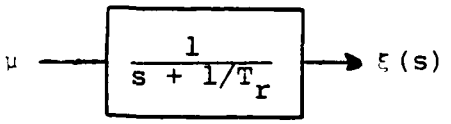

Figure 4-1 Exponentially correlated noise shaping filter

The standard deviation, σ, of the sensor error is a function of the sensors. The values of σ and T_r were obtained from a study by Analytical Mechanics Association (AMA), reference [10], in which an aircraft using a SRFIMF controller was studied. Table 4-1 is a list of the standard deviation estimates which were obtained from the AMA study. Also based upon the AMA study, we assume a value of T_r to be 10 seconds for both sensor models.

	Strapdown	Inertial
Attitude position, σ_p	1°	0.1°/
Attitude rate, σ_v	0.5°/sec	0.5°/sec
Attitude acceleration, σ_a	$1^\circ/sec^2$	$0.1^\circ/s^2$

(T_r = 10 seconds in all cases)

Table 4-1 Sensor error model parameters

The position controller shown in figure 2-4 is augmented with the sensor error sources and the resulting system is shown in figure 4-2.

To analyze the closed loop controller, we first obtain the state space representation of the plant given by equation 4-1. Taking position and velocity as the state variable of the plant we write

$$\text{Position} = X_1 \qquad 4\text{-}4a$$

$$\text{Velocity} = \dot{X}_1 = X_2 \qquad 4\text{-}4b$$

$$\text{Acceleration} = \dot{X}_2 = -cX_1 - bX_2 + CPGX_3 \qquad 4\text{-}4c$$

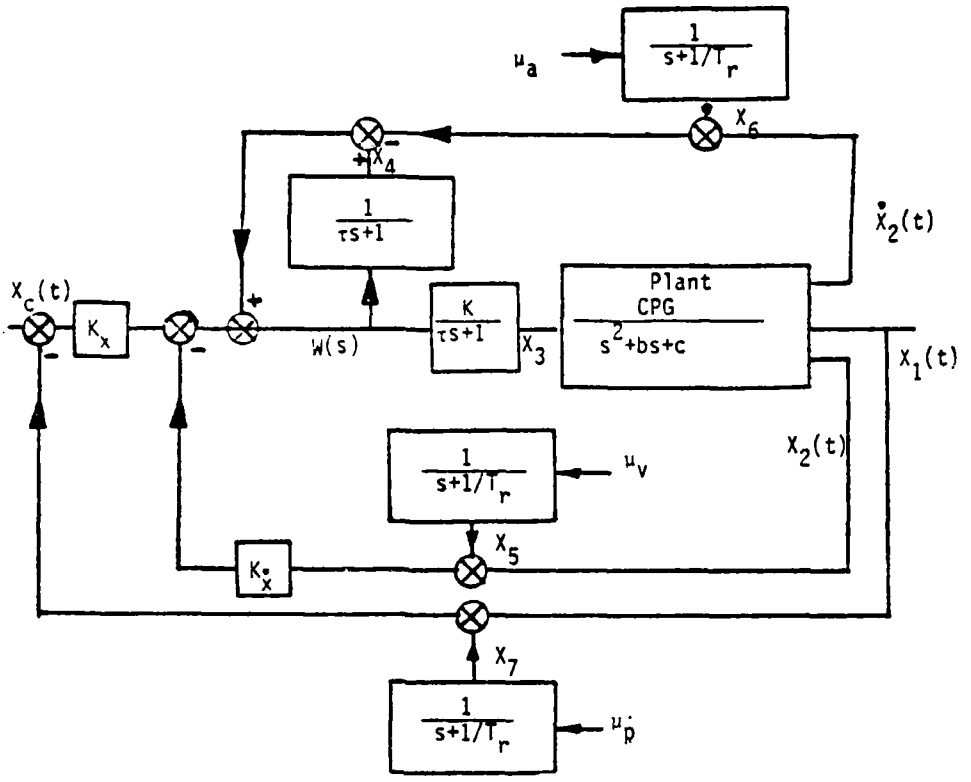

Figure 4-2 High frequency error model, position controller

The additional state variables required for analysis of the controller as shown in figure 4-2 are

X_3 = output of the actuator which is also input to the plant

X_4 = output of the actuator compensator

X_5 = state variable representing the error in the velocity measurement

X_6 = state variable representing the error in acceleration measurement

X_7 = state variable representing the error in position measurement

From the figure we can write the control law W(s), with $X_c = 0$ as

$$W(t) = -K_x(X_1 + X_7) - K_{\dot{x}}(X_2 + X_5) - (-cX_1 - bX_2 + CPGX_3 + X_6) + X_4$$

$$= (-K_x + c)X_1 + (-K_{\dot{x}} + b)X_2 - CPGX_3 + X_4 - K_{\dot{x}}X_5 - X_6 - K_xX_7 \qquad 4\text{-}5$$

The remaining first order differential equations, obtained from figure 4-1, are given as

$$\dot{X}_3 = -1/\tau X_3 + K/\tau W(t) \qquad 4\text{-}6a$$

$$\dot{X}_4 = -1/\tau X_4 + 1/\tau W(t) \qquad 4\text{-}6b$$

$$\dot{X}_5 = -1/T_r X_5 + 2\sigma_v^2/T_r \qquad 4\text{-}6c$$

$$\dot{X}_6 = -1/T_r X_6 + 2\sigma_a^2/T_r \qquad 4\text{-}6d$$

$$\dot{X}_7 = -1/T_r X_7 + 2\sigma_p^2/T_r \qquad 4\text{-}6e$$

Combining equation 4-4, 4-5 and 4-6 and expressing the system in matrix form*, we have the closed loop system whose inputs are the random sensor noise quantities given by

$$\dot{X} = [A]\,X + [B][Q]$$

$$Y = [C]\,X \qquad 4\text{-}7$$

where σ_p, σ_v, and σ_a are the standard deviation of the position measurement error of the velocity measurement error and of acceleration measurement error respectively and [Q] is the matrix of input white noise powers given by

$$Q_{ii} = \mu_i = 2\sigma_i^2/T_r$$

*Matricies A, B, Q and C are shown on following page.

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ (c - K_x)\frac{K}{\tau} & (b - K_x)\frac{K}{\tau} & -(CPG \cdot K + 1)/\tau & \frac{K}{\tau} & -\frac{K_x \cdot K}{\tau} & -\frac{K}{\tau} & -\frac{K \cdot K_x}{\tau} \\ (c - K_x)/\tau & (b - K_x)/\tau & -CPG/\tau & 0 & -\frac{K_x}{\tau} & -\frac{1}{\tau} & -\frac{K_x}{\tau} \\ 0 & 0 & 0 & 0 & -1/Tr & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1/Tr & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1/Tr \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad Q = \begin{bmatrix} 2\sigma_{v/Tr} & 0 & 0 \\ 0 & 2\sigma_{a/Tr} & 0 \\ 0 & 0 & 2\sigma_{p/Tr} \end{bmatrix}$$

$$C = \begin{bmatrix} -c & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & -b & 0 & 0 & 1 & 0 & 0 \\ -c & -b & CPG & 0 & 0 & 1 & 0 \end{bmatrix}$$

Having the matrix representation of the closed loop controller being driven by the sensor noise inputs, we can analyze the effect of the sensor error by considering the controllability and observability of the system with respect to the noise inputs and outputs of position, velocity and acceleration.

The linear control system computer programs, developed by Melsa and Jones [7], were first used with the result that the system is both observable and controllable. In order to gain a better understanding of the controllability and observability of the system, a second approach, following a method suggested by Bryson [11], was used. In the later method we decouple the system of equations by diagonalizing the system given by equation 4-7. This produces a system of equations in modal coordinates. The diagonalization procedure requires that we calculate the eigenvalues and eigenvectors of the A matrix. The transformation matrix, T, from the original coordinates to the new coordinates, is obtained by normalizing, in a complex sense, the matrix of eigenvectors. The matrix quantities corresponding to the modal coordinates are denoted with a superscript, *. The system, in modal coordinates is expressed as

$$X^* = A^*X^* + B^*Q \qquad 4\text{-}8a$$

$$Y = C^*X^* \qquad 4\text{-}8b$$

and the transformation is as follows

$$X^* = T^{-1}X \qquad 4\text{-}9a$$

$$A^* = T^{-1}AT \qquad 4\text{-}9b$$

$$B^* = T^{-1}B \qquad 4\text{-}9c$$

$$C^* = CT \qquad 4\text{-}9d$$

We will now apply the above coordinate transformation to examine the observability and controllability of the system when the plant is a specific second order system.

For this example we chose one of the representative VSTOL second order plants given by mode number 6 of table 3-2. The transfer function for this plant is given by

$$G(s) = \frac{1}{s^2 + .6s + 2.16} \qquad 4\text{-}10$$

We will assume the following additional parameters

$$\tau = .1 \text{ sec}, \; K_x = 4, \; K_x = 3, \; K = 5, \; T_r = 10 \text{ sec}$$

with these choices the gain parameter KRL is

$$KRL = K \; CPG/\tau = 50$$

Because the system is linear with respect to the noise inputs we can solve the problem in general assuming a unit value for the individual noise standard deviations, or

$$\sigma_p = \sigma_v = \sigma_a = 1 \quad .$$

where σ_p, σ_v and σ_a is the standard deviation of the position, velocity and acceleration measurement error. The original system and the resulting diagonalized system was determined by a fortran computer code called MODAL, listed in Appendix A. The results are given

$$X = AX + BQ \qquad X^* = A^*X^* + B^*Q$$
$$Y = CX \qquad Y = C^*X^*$$

Matricies A, B, C, A*, B* and C* are shown on the following pages.

$$A = \begin{bmatrix} 0.0 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ -2.16 & -0.6 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ -92.0 & -120.0 & -60.0 & 50.0 & -150.0 & -50.0 & -200.0 \\ -18.4 & -24.0 & -10.0 & 0.0 & -30.0 & -10.0 & -40.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & -0.1 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -0.1 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -0.1 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \\ 1.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 1.0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 \\ 0.0 & 1.0 & 0.0 & 0.0 & 1.0 & 0.0 & 0.0 \\ -2.16 & -0.6 & 1.0 & 0.0 & 0.0 & 1.0 & 0.0 \end{bmatrix}$$

$$A^* = \begin{bmatrix} -1.56 & 1.34 & 0.0 & -0.0 & 0.0 & -0.0 & 0.0 \\ -1.34 & -1.56 & -0.0 & 0.0 & -0.0 & -0.0 & -0.0 \\ -0.0 & -0.0 & -47.48 & 0.0 & 0.0 & 0.0 & 0.0 \\ -0.0 & 0.0 & -0.0 & -10.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & -0.1 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -0.1 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -0.1 \end{bmatrix}$$

$$B^* = \begin{array}{c} \begin{matrix} Q_v & Q_a & Q_p \end{matrix} \\ \begin{bmatrix} 15.48 & 5.16 & 20.64 \\ 17.91 & 5.97 & 23.88 \\ -3.41 & -1.14 & -4.54 \\ 0.0 & 0.0 & 0.0 \\ 2.17 & 0.0 & 0.0 \\ 0.0 & 1.19 & 0.0 \\ 0.0 & 0.0 & 2.75 \end{bmatrix} \end{array}$$

$$C^* = \begin{array}{c} \begin{matrix} & & & & x_v^* & x_a^* & x_p^* \end{matrix} \\ \begin{bmatrix} 0.05 & 0.0 & -0.0 & 0.01 & -0.87 & -0.23 & -0.03 \\ 0.08 & 0.07 & 0.02 & -0.08 & 0.5 & 0.02 & 0.04 \\ 0.03 & -0.22 & -0.99 & 0.82 & -0.0 & 0.84 & -0.0 \end{bmatrix} \end{array}$$

It is instructive to observe the A matrix of the system. In the upper right corner, rows and columns 1 and 2 represent the plant. Row 3 is derived from the controller terms, $K \cdot W(t)/\tau$. These terms dominate the matrix.

The diagonal elements of A* are the eigenvalues of the original system. All of the eigenvalues are negative, indicating that the system is stable. The system has one coupled oscillatory mode, -1.56, 1.34 in the upper right corner of A*. This mode corresponds to the model and will be referred to as the model mode throughout the remainder of this study. The eigenvalue -47.48, row 3 column 3 of A*, is the mode which represents the output of the controller. We refer to this as the controller mode. It physically represents the real pole whose location corresponds to the value of KRL as discussed in the root locus analysis of section III-C. Similarly the eigenvalue -10.00, row 4 column 4 of A*, is the compensator mode. The other three eigenvalues correspond to the noise filter time constant, $-1/T_r$. From the transformed system, the controllability and observability of each of the previously mentioned modes with respect to the noise input can be determined.

In the B* matrix the Q_v above the first column indicates that column 1 is the input vector corresponding to velocity error measurement input. The other columns are annotated similarly. The first two rows of the B* matrix indicate that the model and controller modes are affected by all three

measurement errors and that acceleration measurement error is least significant. The zeroes in row 4 of the B* matrix indicate that the compensator mode is unaffected by measurement noise. This indicates that the compensator does not contribute to the uncertainty of the system.

The C* matrix has been annotated in a way similar to B* so the X_v^*, X_a^*, and X_p^* correspond to the velocity, acceleration, and position error modes respectively. From the C* matrix it is seen that the observation of position, corresponding to the first row of the matrix, is affected by all three measurement errors but that velocity and acceleration are only slightly affected. We conclude from the C* matrix that measurement errors will affect the position, which is the quantity being controlled, but that the dynamic behavior, velocity and acceleration are only slightly affected. This result is perhaps due to the low frequency error model which results from the choise of T_r equal to 10 seconds.

From the modal analysis it is concluded that sensor errors can affect the model and controller mode of the closed loop system and that the errors can be observed in the measurement of the position. The exact relationship between the output quantities of position, velocity and acceleration will be determined in the next section where the variances of state variable X_1, X_2, and X_3 will be found by solving the Lyapunov equation for the system.

B. COVARIANCE ANALYSIS

The object of the covariance analysis is to determine the standard deviations of the error in the states X_1, X_2, and X_3 of the system developed in the previous section. The standard deviation of X_1 and X_2 are of interest because they represent position and velocity tracking accuracy. X_3 is of interest because it is the input to the plant. A large deviation in X_3 due to measurement error would mean unnecessarily high amounts of control energy lost because of measurement errors. The state variable representation of the system developed in section 4-1, where the inputs to the system are noise sources representing position, velocity and acceleration measurement error, will again be used.

The problem of determining the covariance of the states of a system in the presence of disturbances is typically encountered in the design of optimal estimators, the Kalman filter. In the estimator problem the system is expressed in the following way

$$\dot{X} = AX + BU + GW_t(t) \qquad 4\text{-}11$$

where A is the matrix representation of the plant, B the control input matrix, $W_t(t)$ white noise disturbance to the plant states with

$$E[W_t(t)] = 0 , \qquad E[W_t(t), W_t^T(t)] = \mu$$

and G the input matrix of the disturbances. In estimation problems one assumes that measurements of the system are made

which can be expressed in terms of the states of the system by the relationship

$$\underline{Z}_N = H\underline{X} + \underline{V}_N$$

Here N quantities are measured with an expected error of V_N.

In the developement of the SRFIMF controller, sensor noise was defined as a state of the system and the A matrix was augmented to include these states. The covariance analysis assumes quiescent operation of the controller, therefore, the only input to the system is white noise of strength, μ, given by equation 4-3. The white noise acts only to disturb the sensor error states. The governing equation for the covariance of the states is given by the Lyapunov differential equation

$$P = AP + PA^T + GQG^T \qquad 4\text{-}12$$

where P is the matrix of the state variable covariances defined as

$$P_{ij} = \sigma_i \sigma_j$$

and Q is as before, the matrix of white noise input whose power is μ. We have chosen to rename the matrix G of equation 4-11 to B because the white noise sources are considered to be the inputs to the system.

Again a unit value of standard deviations was assumed and, because the system is linear with respect to the noise inputs, we can apply later the set of sensors. We will consider the

same plant as used in section 4-1. The transfer function for the plant is given by

$$\frac{G(s)}{U(s)} = \frac{CPG}{s^2 + .6s + 2.16} \qquad 4\text{-}13$$

At time equal to zero no uncertainties exist in the system, therefore the initial condition on P is zero.

A computer program, Vary, listed in Appendix A, was developed to solve the Lyapunov equation. The program uses an International Mathematical and Statistical Library (IMSL) subroutine called DVERK. This subroutine solves a system of first order differential equations using a Runge-Kutta Algorithm. The resulting covariance matrix, P, is printed by the program as a function of time, at a number of discrete times. The diagonal elements of the P matrix are interpreted as the square of the standard deviation or RMS value (σ_{ii}^2) of each of the state variable uncertainties as a result of the noise input. The data obtained from the computation of the P matrix is plotted in figures 4-3 to 4-5 and tabulated in Appendix B, table B-1. The figures show the standard deviation (sigma) of the states X_1, X_2 and X_3 as functions of time from zero to ten seconds. When necessary, the steady state value is shown by a "+" on the figure.

Figures 4-3 to 4-5 can be interpreted in the following way. Each individual curve represents the contribution to the total state variable error as a result of one measurement

source. For example, in figure 4-3 we see that a unit value of standard deviation error in position measurement results in a standard deviation error in X_1 of approximately 0.8° at six seconds, and a steady state error of 0.98°. The total expected value of the uncertainty in X_1 is the sum of the errors resulting from position, velocity and acceleration measurements as given by

$$\sigma_i = \sigma_{ip} + \sigma_{iv} + \sigma_{ia} \qquad 4\text{-}14$$

Where the notation σ_{ij} refers to the ith state variable and j refers to a noise source. Thus, σ_{ip} refers to the standard deviation of X_1 as a result of position measurement error. Table 4-2 summarizes the steady state values of the components of the state variable errors.

	X_1	X_2	X_3
Position measurement error	.980	.360	2.13
Velocity measurement error	.735	.270	1.60
Acceleration measurement error	.245	.089	0.53

Table 4-2 Steady state errors as a result of measurement errors

The actual measurements are made either by inertial navigation sensors or by strapdown sensors whose standard deviations are given by table 4-1. We can now apply the results obtained from the covariance analysis to obtain an estimate of the state variable errors for actual measurement

cases. For example, the steady state standard deviation of the error in X_1 when the measurements are made by an inertial system is given as

$$\sigma_1 = q_p\sigma_{1p} + q_v\sigma_{1v} + q_a\sigma_{1a} \qquad 4\text{-}15$$

where q_p, q_v and q_a are the standard deviations of the inertial sensor measurements given by table 4-1 as

$$q_p = 0.1^\circ$$
$$q_v = 0.5^\circ/\text{sec}$$
$$q_a = 0.1^\circ/\text{sec}^2$$

and σ_{1p},σ_{1v} and σ_{1a} are given in table 4-2. The resulting expected error in X_1 in steady state is found from equation 4-15 to be 0.49°. In a similar way, the steady state errors in X_1, X_2 and X_3 for both inertial measurements and strapdown measurements can be evaluated. The results are shown in table 4-3.

	X_1	X_2	X_3
Inertial sensors	0.49°	$0.18^\circ/\text{sec}$	$1.07^\circ/\text{sec}^2$
Strapdown sensors	1.6°	$0.58^\circ/\text{sec}$	$3.46^\circ/\text{sec}^2$

Table 4-3 Steady state tracking errors produced by strapdown and inertial navigation sensors

The result of the covariance analysis indicates that the expected uncertainity in X_1 and X_2 as a result of measurement errors are not significant. For example, in the case of strapdown sensors, the expected error in position, X_1, is 1.6°

while measurement errors had standard deviation values of 1°, $0.5^{\circ}/sec$ and $1^{\circ}/sec^2$.

To interpret the significance of the expected value of the error in X_3, the value of X_3 will be determined when a 1° input is applied to the controller. Refer to figure 4-2 and, for this example, disregard the effect of the control actuator at the input to the plant. Assuming that initially the controller is in steady operation with feedbacks equal to zero, at the instant the 1° input is applied the value of X_3 is determined to be

$$X_3 = X_c \cdot K_x \cdot K = 20 \qquad 4\text{-}16$$

Comparing the value of the standard deviation in X_3 to the value of X_3 when a 1° unit step input is applied to the controller, we conclude that the value of the standard deviation error in X_3 is only 17.3 percent of the value of X_3 when a 1° input is applied to the system.

From both the modal analysis and the variance analysis it can be concluded that high quality acceleration measurements are not necessary and that errors in the measurement of position and velocity are not amplified. The errors in the states are most affected by position measurement error. This result might have been expected because the controller is attempting to track to the commanded position and errors in the position measurement should dominate the uncertainty in the other states. Figures 4-3 thru 4-5 show that the uncertainty in the states reaches a

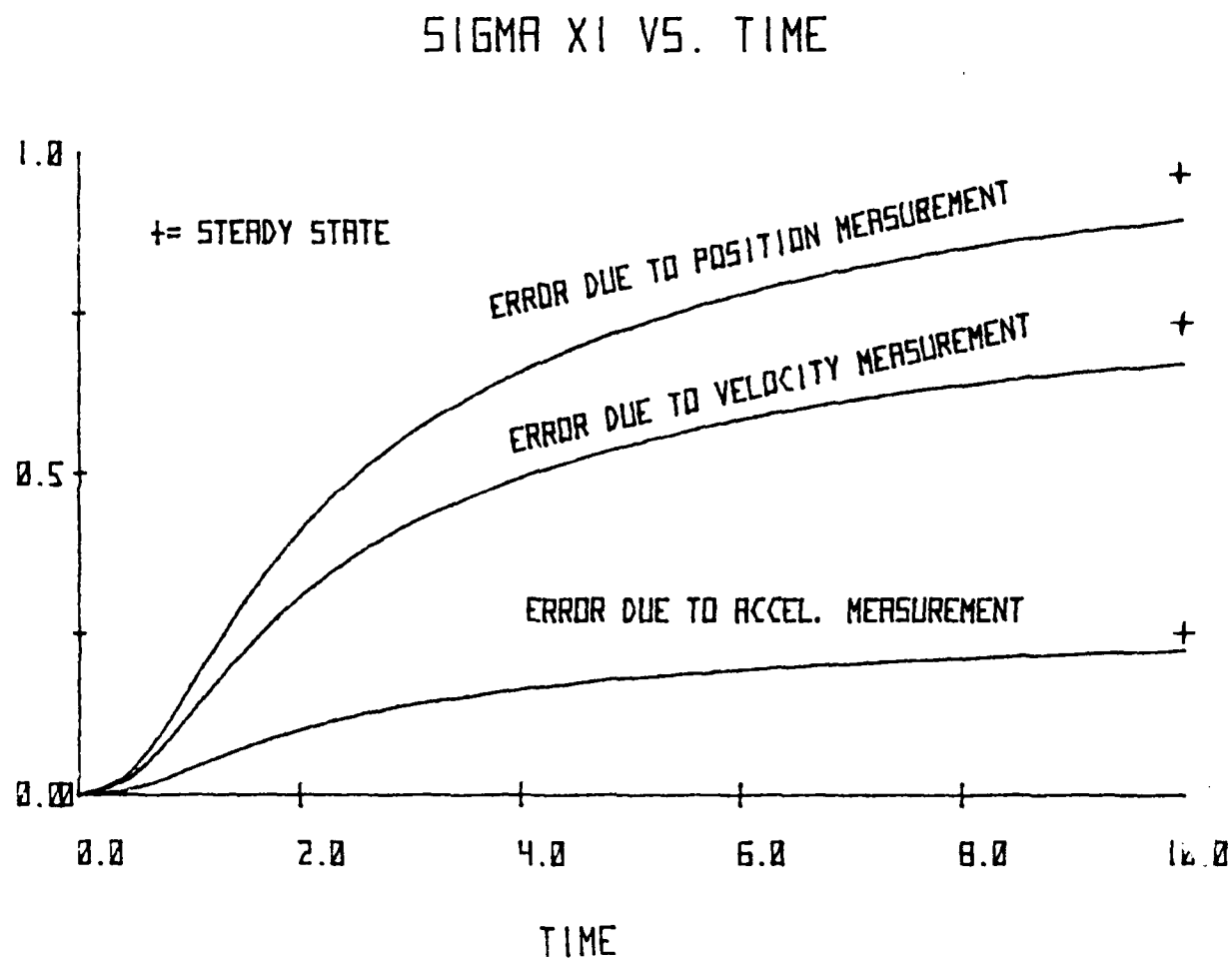

Figure 4-3 Tracking errors in X_1 as a result of measurement error in position, velocity and acceleration.

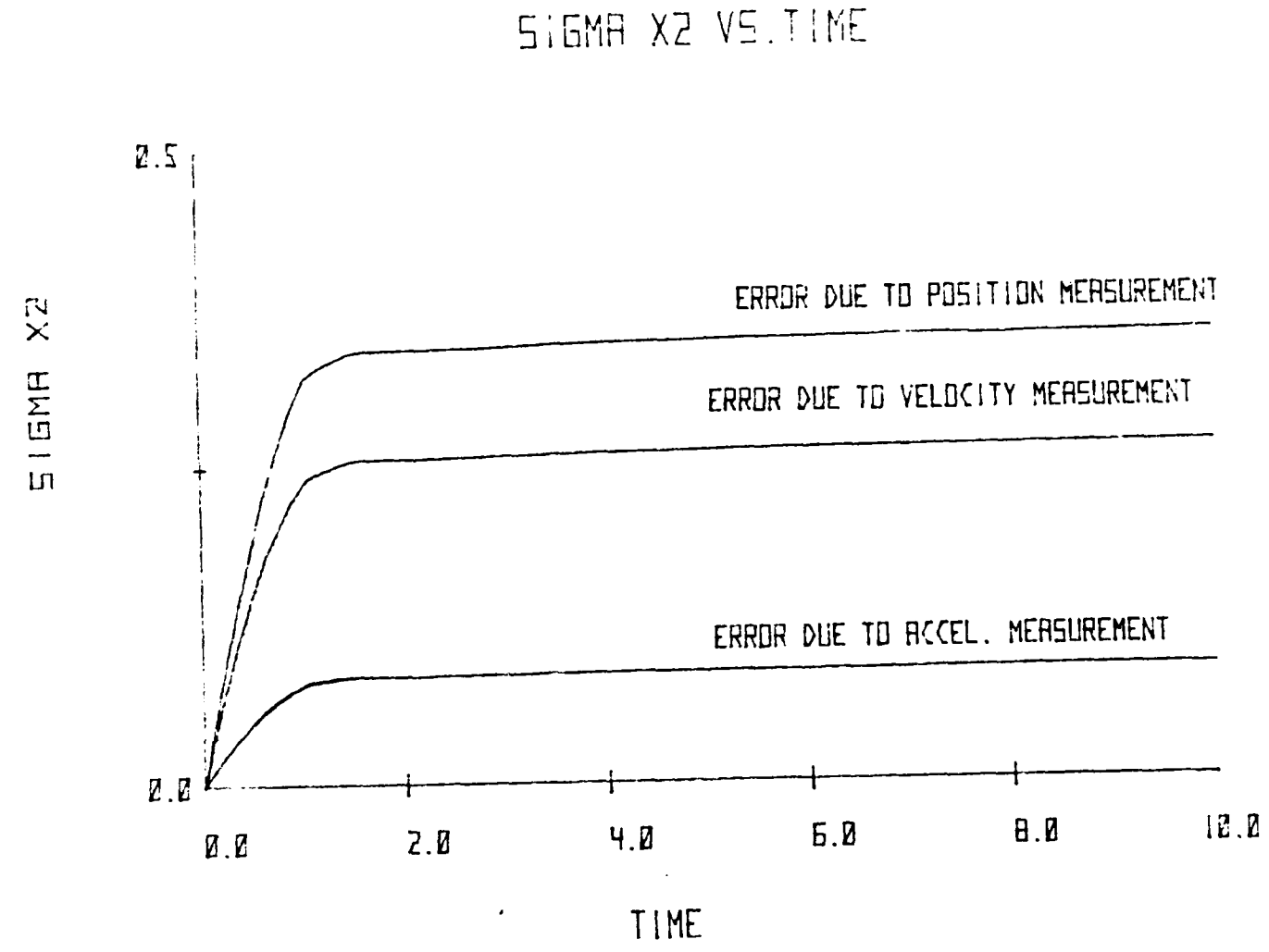

Figure 4-4 Tracking errors in X_2 as a result of measurement error position, velocity and acceleration.

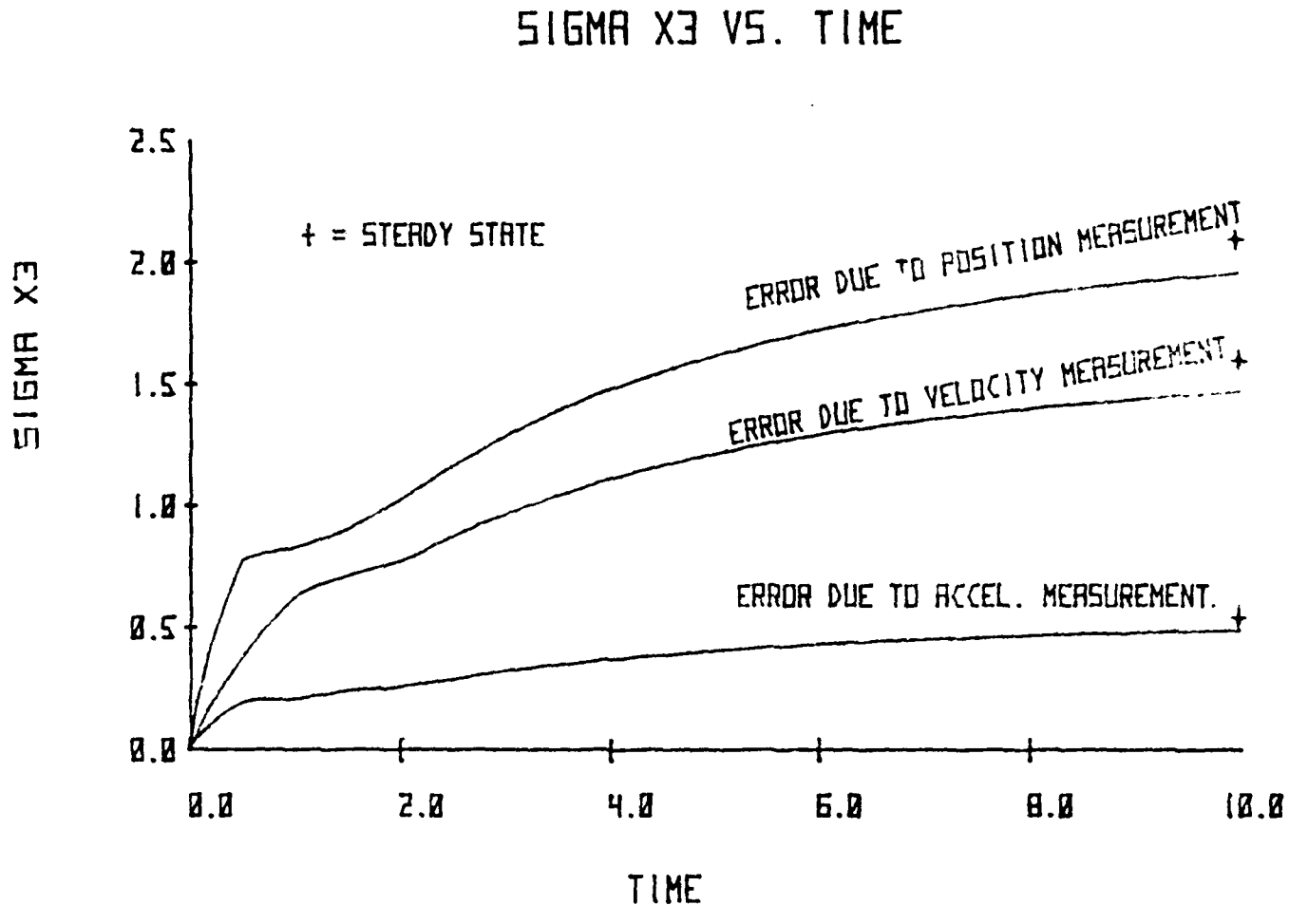

Figure 4-5 Tracking errors in X_3 as a result of measurement error in position, velocity and acceleration.

near steady state value in a short time, particularly in the case of X_2. The rise time of the errors, seen in the figures, should not be interpreted as a lag in the response, since the figures represent the expected error in each of the states.

C. ALTERNATE MEASUREMENT SCHEMES

Two possible methods of reducing the number of measurements required by the SRFIMF controller will be considered. The first case is to measure acceleration and integrate to obtain velocity and position. In the second scheme, position and velocity will be measured and acceleration will be estimated from knowledge of the plant. The second case will be used to obtain an estimate of the added uncertainty caused by the acceleration measurement. Figure 4-6 is a schematic representation of the first case. As before the state representation of the system is obtained.

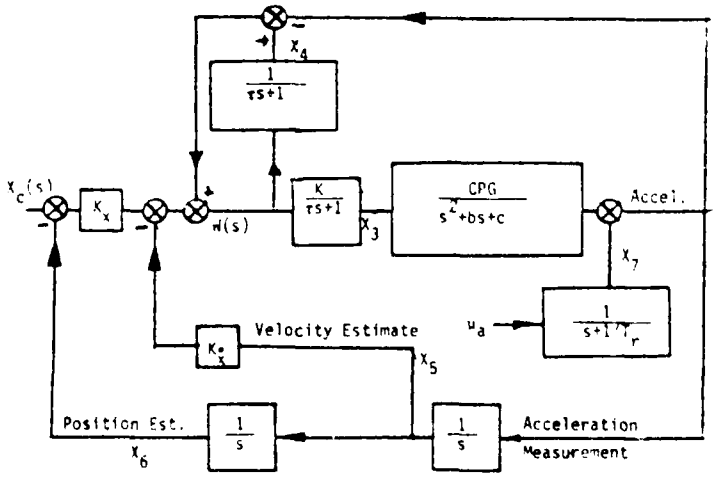

Figure 4-6 First alternate measurement scheme. Measured acceleration and implied position and velocity.

The state variables are defined as

X_1 = position

X_2 = velocity

X_3 = output of the control actuator

X_4 = output of the compensator

X_5 = velocity estimate

X_6 = position estimate

X_7 = acceleration error

The control input, from figure 4-5 is written as

$$W(s) = cX_1 + bX_2 - CPGX_3 + X_4 - K_xX_5 - X_7 \qquad 4\text{-}16$$

Following a development analogous to that used in section 2-3 we obtain the state representation of the system to be

$$\dot{X} = AX + BQ$$

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -c & -b & CPG & 0 & 0 & 0 & 0 \\ K\cdot c/\tau & b\cdot K/\tau & -(CPG\cdot K+1)/\tau & K/\tau & -K\cdot K_{\dot{x}}/\tau & -KK_x/\tau & -K/\tau \\ c/\tau & b/\tau & -CPG/\tau & 0 & -KX/\tau & -KX/\tau & -1/\tau \\ -c & -b & CPG & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1/T_r \end{bmatrix}$$

$$B = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{Bmatrix} \qquad Q = \begin{bmatrix} \frac{2\sigma_a}{T_r} \end{bmatrix}$$

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We apply the modal computer program to this system using the plant defined by equation 4-10 in order to determine the observability and controllability. The resulting system in modal coordinates is

$$\dot{X}^* = A^* X^* + B^* Q$$

$$A^* = \begin{bmatrix} -47.48 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & -1.56 & 1.34 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & -1.34 & -1.56 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -10.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -0.1 \end{bmatrix}$$

$$B^* = \begin{bmatrix} -1.07 \\ .17 \\ -.20 \\ -211.0 \\ 9989.68 \\ 0.0 \\ 237.0 \end{bmatrix}$$

$$C^* = \begin{bmatrix} 0.0 & -0.03 & -0.05 & 0.46 & 0.0 & -0.01 & -0.42 \\ 0.02 & 0.12 & 0.04 & 0.0 & 0.0 & 0.08 & 0.04 \\ -0.99 & -0.24 & 0.09 & 0.0 & 0.0 & -0.82 & 0.0 \end{bmatrix} \quad X_a^*$$

From the A* matrix we can see that this system has two zero eigenvalues meaning that the modes of the system which correspond to the open loop integrators are neutrally stable. The modal control vector B* indicates that the velocity and position estimate modes of this system are strongly affected by the noise input. In particular, mode five (row 5 of B*) which represents the position estimate mode has a control coefficient from the noise source four orders of magnitude greater than the model mode (rows 2 and 3 of B*). We expect that in this case acceleration sensor noise significantly affects the performance of the controller.

Applying the algorithm used in the previous section to this case we obtain the covariance estimate. Figure 4-7 is a plot of the standard deviation (sigma) of X_1, X_2 and X_3 assuming

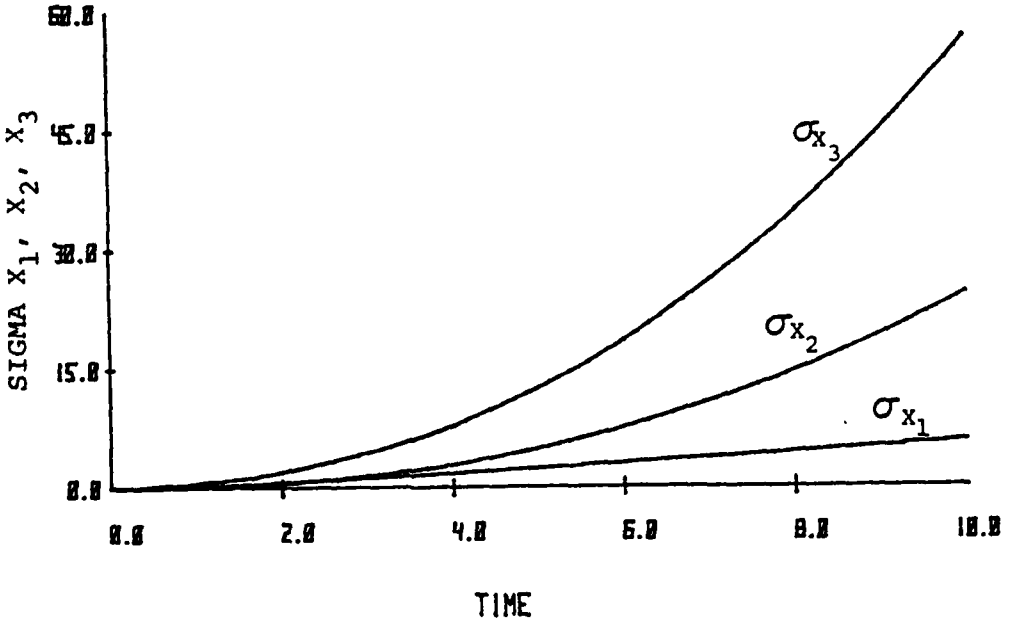

Figure 4-7 Error in X_1, X_2, and X_3 as a result of measurement errors in acceleration.

a unit standard deviation noise source as the acceleration measurement error. Data used to plot figure 4-7 is also tabluated in Appendix B, table B-2. It is seen from the figure that acceleration measurement error causes unbounded errors in X_1 and X_3. Unbounded errors in any of the states are unacceptable in the controller and unless we can improve the method of estimating the position and velocity, we will be required to measure these quantities. A Luenberger observer could be used to estimate position and velocity, however the design of the observer would requre use of knowledge of the plant. We wish to avoid using detailed knowledge of the plant.

The second alternate measurement scheme is the case of measured position and velocity with acceleration estimated from the plant parameters b and c. The estimation is given by

$$\text{Acceleration} = -cX_1 - bX_2 + CPG\ X_3 \qquad 4\text{-}17$$

This scheme is shown in figure 4-8.

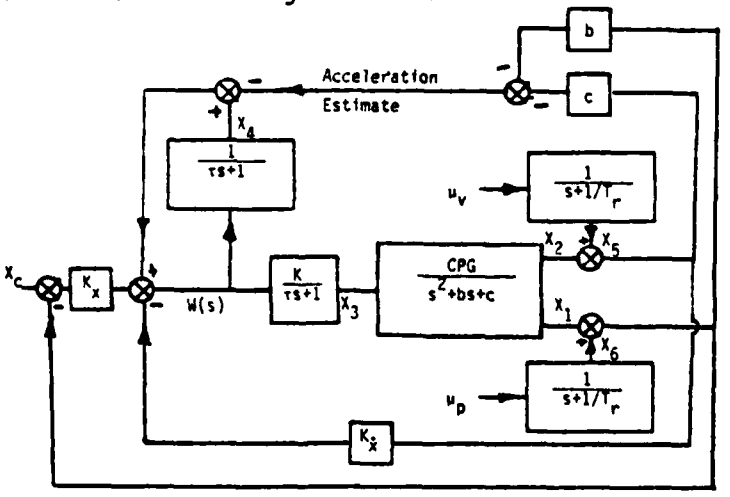

Figure 4-8 Second alternate measurement scheme, measured position and velocity. Estimated acceleration.

Following the same procedures as before we define the state variable as

X_1 = position

X_2 = velocity

X_3 = output of the control actuator

X_4 = output of the compensator

X_5 = position measurement error

X_6 = velocity measurement error

From figure 4-8 we obtain the control law W(t) as

$$W(t) = (-K_x + c)\, X_1 + (-K_{\dot{x}} + b) X_2 - CPG\, X_3 + X_4 + (-K_x + c) X_5 + (-K_{\dot{x}} + b) X_6 \qquad 4\text{-}18$$

Assuming the same second order plant given by equation 4-10 we obtain the modal transformation of the system.[1]

$$X = AX + BQ$$

$$Y = CX$$

$$X^* = A^*X^* + B^*Q$$

$$Y = C^*X^*$$

[1] Matricies A, B, C, A*, B* and C* are shown on the following pages.

$$A = \begin{bmatrix} 0.0 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ -2.16 & -.6 & 1.0 & 0.0 & 0.0 & 0.0 \\ -92.0 & -120.0 & -60.0 & -50.0 & -92.0 & -120.0 \\ -18.0 & 24.0 & -10.0 & 0.0 & -18.0 & -24.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & -.1 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -.1 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.0 & 0.0 \\ 0.0 & 0.0 \\ 0.0 & 0.0 \\ 0.0 & 0.0 \\ 1.0 & 0.0 \\ 0.0 & 1.0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ -2.16 & -.6 & 1.0 & 0.0 & 0.0 & 0.0 \end{bmatrix}$$

$$A^* = \begin{bmatrix} -1.56 & 1.34 & 0.0 & 0.0 & 0.0 & 0.0 \\ -1.34 & -1.56 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & -47.48 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & -10.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & -.1 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -.1 \end{bmatrix}$$

$$B^* = \begin{array}{c} \begin{matrix} Q_v & Q_p \end{matrix} \\ \begin{bmatrix} 9.49 & 12.38 \\ 10.99 & 14.33 \\ -2.09 & -2.73 \\ 0.0 & 0.0 \\ 1.55 & 0.0 \\ 0.0 & 1.83 \end{bmatrix} \end{array}$$

$$C^* = \begin{array}{c} \begin{matrix} & & & & x_v^* & x_p^* \end{matrix} \\ \begin{bmatrix} 0.05 & 0.0 & 0.0 & 0.01 & -0.32 & -0.35 \\ -0.08 & 0.07 & 0.02 & -0.08 & 0.03 & 0.04 \\ 0.03 & -0.22 & -0.99 & 0.82 & 0.0 & 0.0 \end{bmatrix} \end{array}$$

The results of the modal analysis are similar to those obtained when acceleration was a measured quantity rather than calculated as in this case. The model and controller modes are controllable from the noise sources. The position and velocity outputs (rows 1 and 2 of the C* matrix) contain sensor noise terms.

The covariance analysis results are shown in figures 4-9 to 4-11 and listed in Appendix B, table B-3. Here it is seen that the standard deviation of X_1, X_2 and X_3 is slightly less than in the case when acceleration was measured. This is because the quantity taken to be acceleration does not contain the additional error of actual acceleration measurement. In this case the system requires only measurements of position and velocity.

The second alternate measurement scheme can be used to compare a state rate feedback control scheme to a state variable feedback controller, from the viewpoint of increased uncertainty in the state variable resulting from the additional measurement of acceleration. To make the comparison, assume that the measurements are made by strapdown type sensors whose measurement errors are given in table 4-1. Table 4-4 lists the total uncertainty of X_1, X_2 and X_3 for both measurement schemes. It can be seen from table 4-4 that the overall uncertainty of X_1, X_2 and X_3 is increased when acceleration is measured, as compared to the second alternate measurement scheme when acceleration is obtained without measurement. In a practical

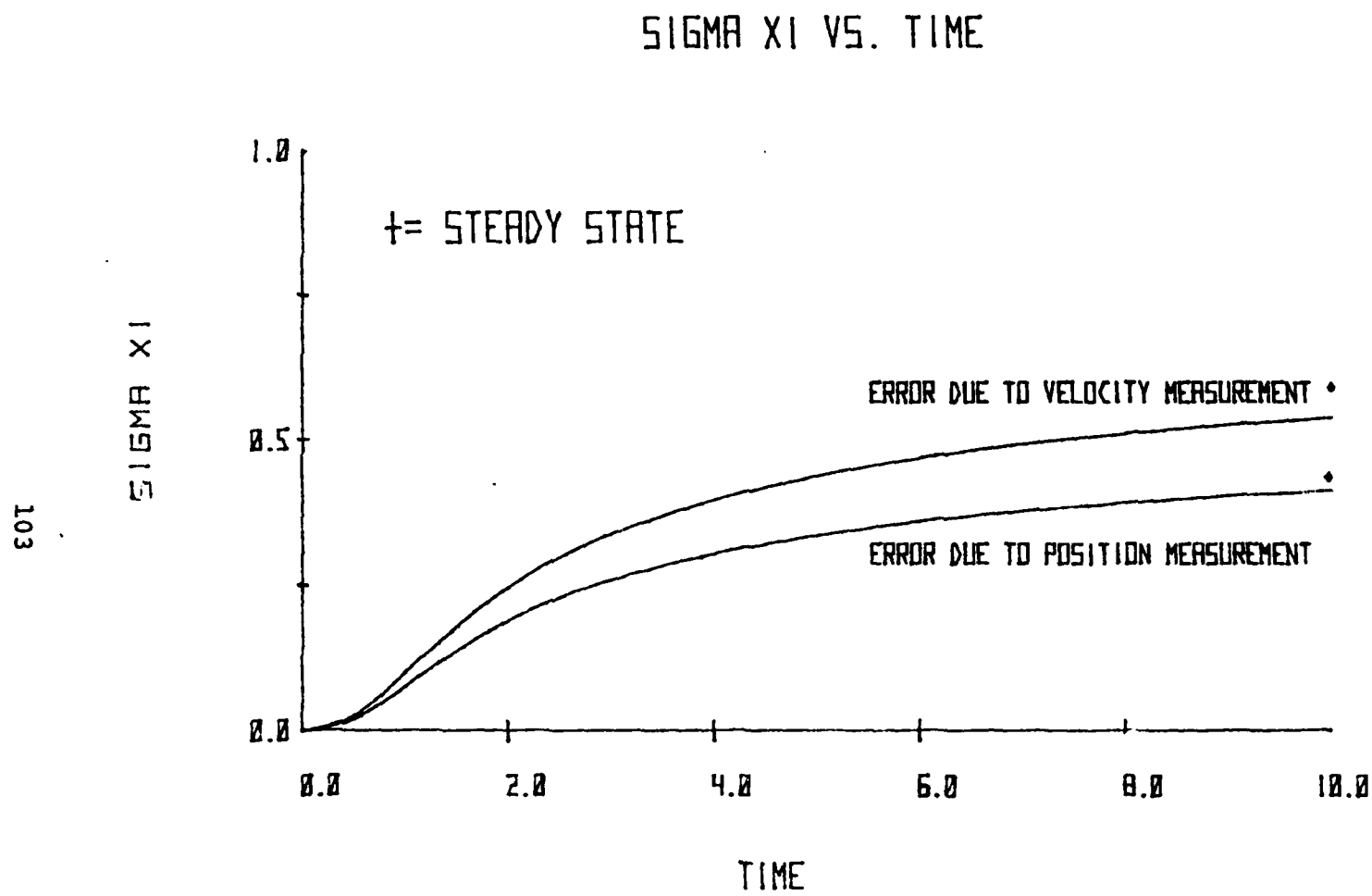

Figure 4-9 Tracking errors of X_1 as a result of measurement errors in position and velocity.

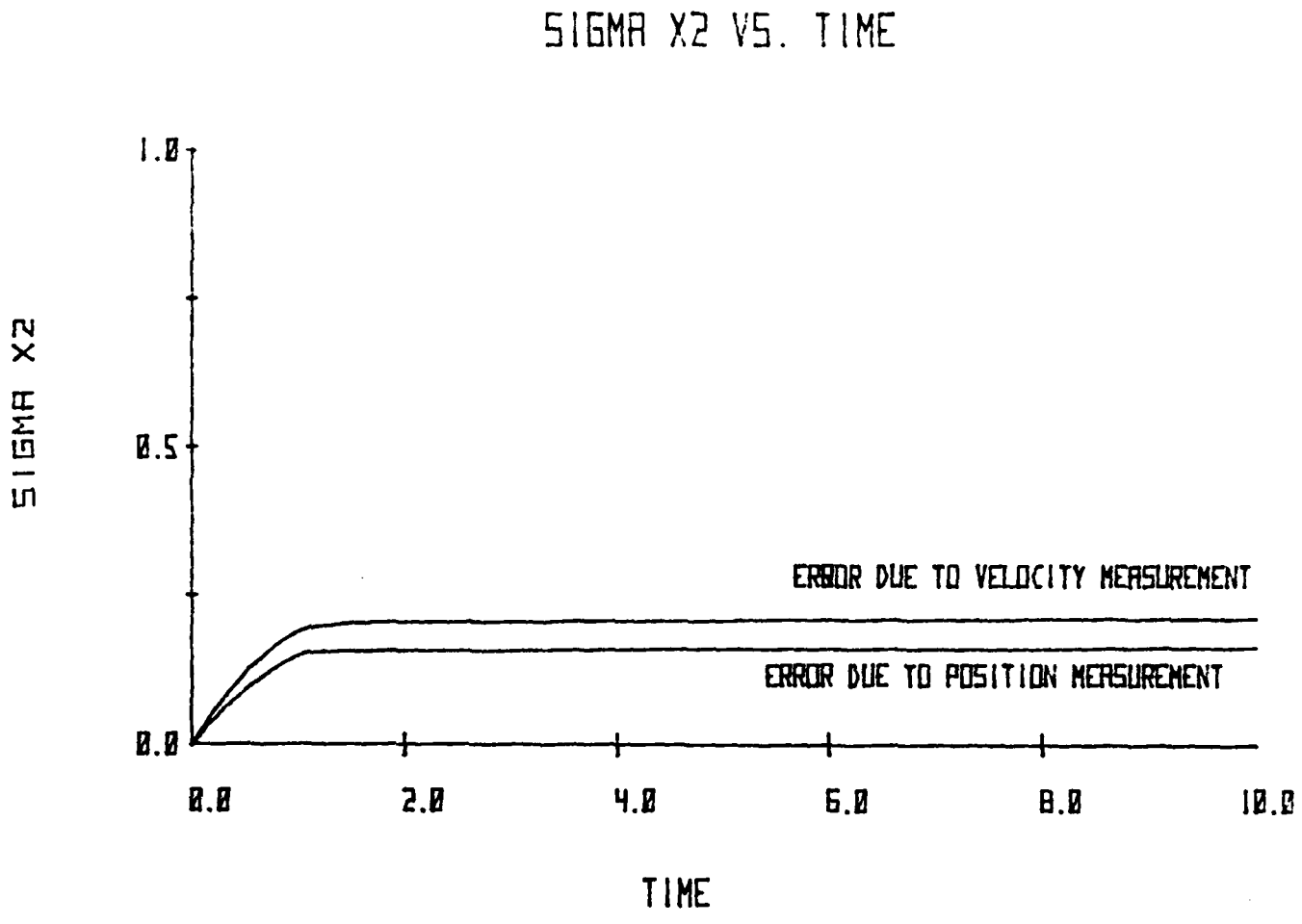

Figure 4-10 Tracking errors in X_2 as a result of measurement errors in position and velocity.

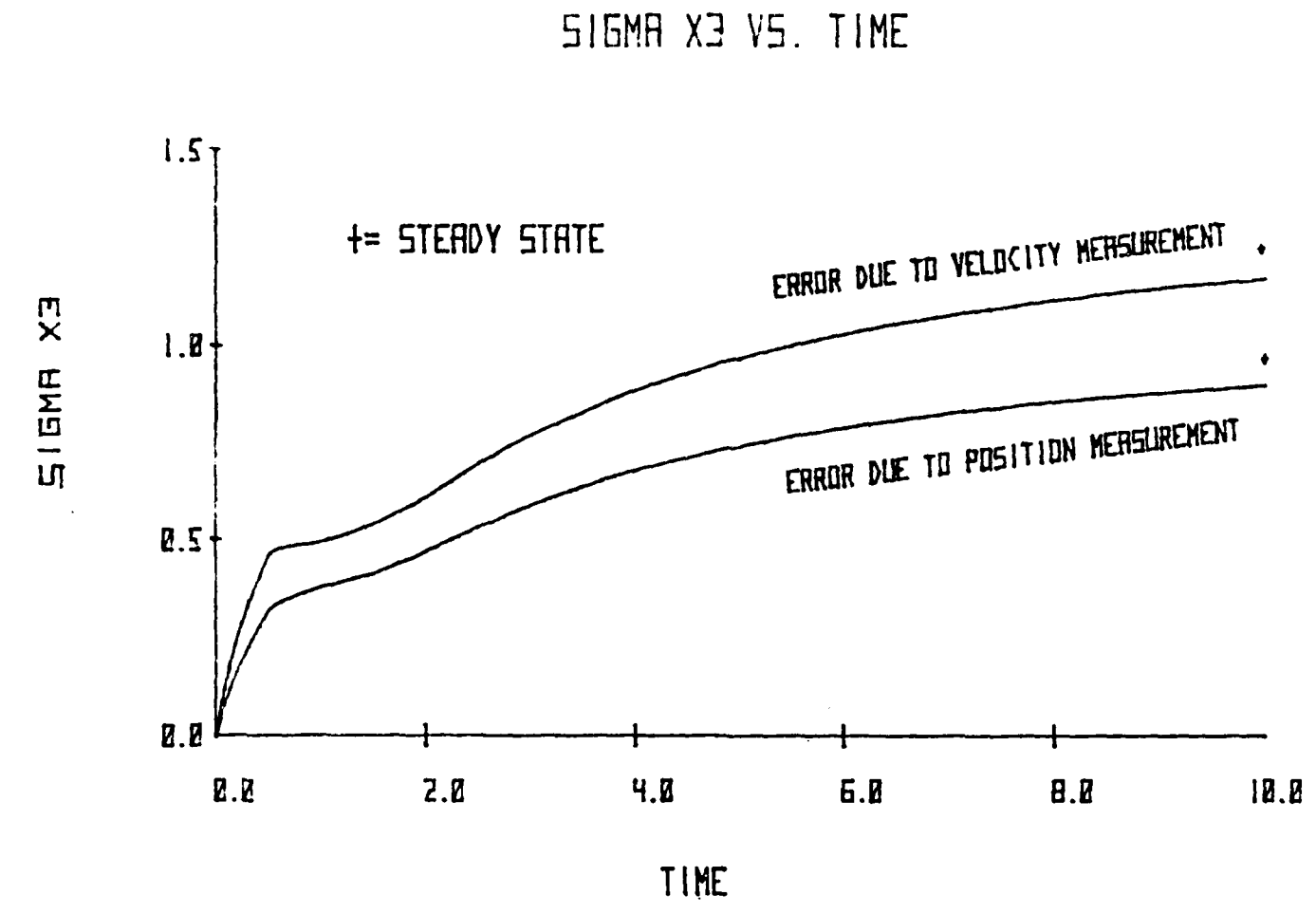

Figure 4-11 Tracking errors in X_3 as a result of measurement errors in position and velocity.

	X_1	X_2	X_3
Second Alternate Measurement Scheme, Calculated Acceleration	0.75°	0.267°/s	1.6°/s^2
Measured Acceleration	1.6°	0.58°/s	3.46°/s^2
Percent increase of expected error	113%	117%	116%

Table 4-4 Steady state error in X_1, X_2 and X_3 for measurements made by strapdown sensors.

system, uncertainty would exist in the knowledge of constants b and c and it could be expected that this would result in additional uncertainty in the value of the state variables. It might be possible that the uncertainty in the knowledge of the plant could negate the advantage gained by estimating acceleration as in the second alternate measurement scheme.

In conclusion, it has been shown that sensor noise does not adversely affect the SRFIMF position controller. It was also shown that acceleration measurement alone is not sufficient for acceptable operation of the controller. The analysis technique used in this section will next be applied to the analysis of a SRFIMF controller when the plant is assumed to be the longitudinal axis of the Harrier aircraft.

V. APPLICATION OF THE SRFIMF CONCEPT TO THE HARRIER AIRCRAFT

A. PITCH ATTITUDE CONTROL

The preceeding discussion dealt with the SRFIMF controller with the plant assumed to be a general second order system. In this section, the closed loop system will be modeled using the transfer function of a VSTOL aircraft. For this purpose the AV-8A Harrier, a jet-lift type VTOL aircraft, was chosen. The stability derivatives and transfer functions were obtained from reference [12]. The SRFIMF controller concept will be applied to the pitch attitude control of the aircraft and the analysis will be similar to that done earlier when the plant was assumed to be that of a second order system. As before, we will examine the root locus, time response and the effect of measurement errors. The effect of gust inputs to the plant will also be considered.

The transfer function between the pilot's stick and the pitch attitude of the Harrier, at 60 kts, 100 ft/sec, as given by reference [12] is

$$\frac{\theta(s)}{\delta_e(s)} = \frac{0.25\ (s^2 + .246s + .00756)}{s^4 + .4896s^3 - .4495s^2 + .06736s + .00747} \qquad 5\text{-}1a$$

$$= \frac{0.25\ (s + .21)(s + .036)}{(s + 1)(s + .073)(s^2 - .1864s + .1024)} \qquad 5\text{-}1b$$

$\theta(s)$ is given in radians and $\delta_e(s)$ in inches of stick displacement. The above transfer function does not include the effect of the control actuator which we include, as before, in the model of the controller.

The denominator factors of equation 5-1b indicate that at 60 kts the Harrier has an unstable oscillatory mode with a natural frequency, of 0.32 and damping ratio of -0.91, and that the constant plant gain, CPG, is 0.25. The plant, given by equation 5-1, is used to compute the root locus for the system by applying equation 3-21, developed for the controller in section III and rewritten here for convenience

$$1 + \frac{KRL(s^2 + K_{\dot{x}}s + K_x)}{S} G'(s) = 0 \qquad 3\text{-}21$$

where G'(s) is the open loop plant given by

$$G'(s) = \frac{G(s)}{CPG}$$

For the root locus evaluation, KRL was varied from 1 to 100. The value of the actuator time constant, τ, was chosen to be 0.1 sec and $K_{\dot{x}}$ and K_x were 3 and 4 respectively.

The root locus of the oscillatory pole is given in figure 5-1. The trajectory of this pole is similar to those given in section III where the plant was assumed to be a second order system. As before, the closed loop system oscillatory poles are near those of the model (-1.5, 1.32) at KRL values of about 25 and greater. Because of the scale of the figure,

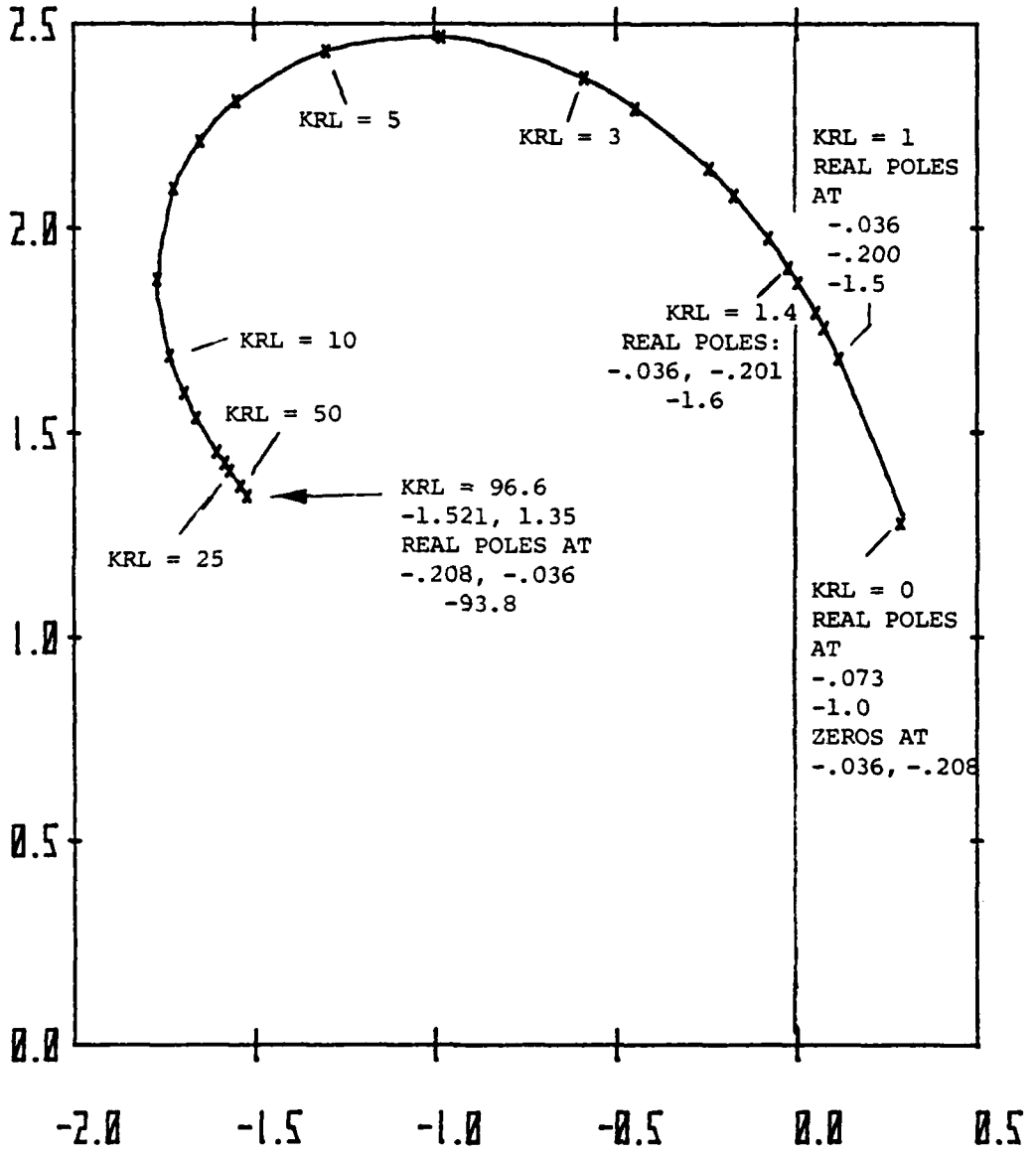

Figure 5-1 Root locus of the oscillatory poles of the longitudinal axis of the Harrier using SRFIMF position control. Note that the open loop zeros are nearly canceled at KRL = 1.0.

the real poles are not plotted, however their values are listed on the figure for various values of KRL. At the final value of KRL (96.6), the system's real poles were located at -93.82, -.2079 and -.036. The pole at -93.28 corresponds to the controller mode and its location is nearly equal to the value of KRL. This result was discussed in section three. The two other poles located at -.036 and -.2079 cancel the open loop zeroes of the plant located at -.036 and -.21 as seen in the open loop transfer function, equation 5-1b. Zero cancellation by a pole could be very detrimental to the response if the open loop zeroes are located in the right half of the complex plane, becaue it would be unreasonable to expect perfect pole-zero cancellation. This is a problem typically encountered in model following techniques when the plant has zeroes in the right half plane.

From the root locus, figure 5-1, it can be seen that the dynamic behavior of the SRFIMF controller is unchanged by the introduction of a more complicated plant. We shall now consider the time response of the closed loop system.

The controller and plant system was simulated using the CSMP program discussed in section III. The source code is listed in Appendix A. Simulation of the Harrier transfer function is illustrated using a signal flow graph shown in figure 5-2. The outputs of position, velocity and acceleration are shown in the figure as part of the overall transfer function simulation. For the simulation, KRL was chosen to be 50, τ was

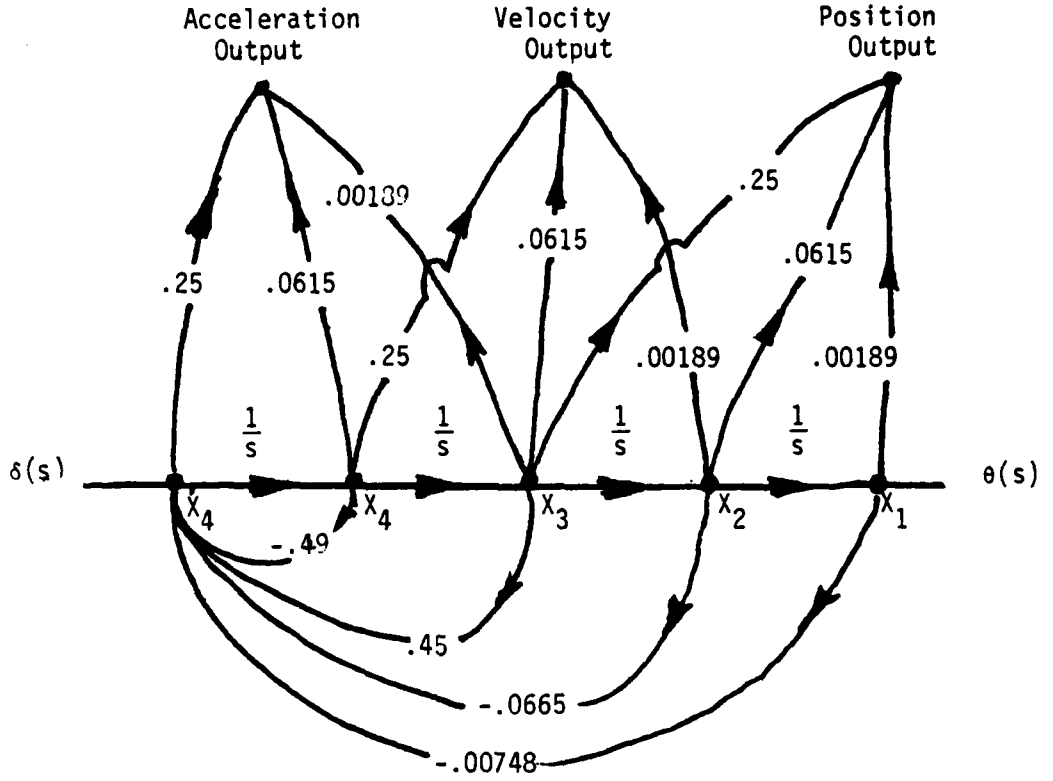

Figure 5-2 Harrier longitudinal signal flow graph transfer function simulation.

0.1 sec, K_x and K_x were 3 and 4. Plots of the time histories of the closed and open loop position, $\theta(t)$, as a result of the simulation, are given in figure 5-3. The figure indicates that the closed loop dynamic response is nearly identical to that of the model as shown in figure 3-5.

To evaluate the effect of the sensor noise, the state space representation of the system is required. Four states represent the plant. Defining X_1 and position and X_2 as velocity we write the transfer function for the Harrier, given

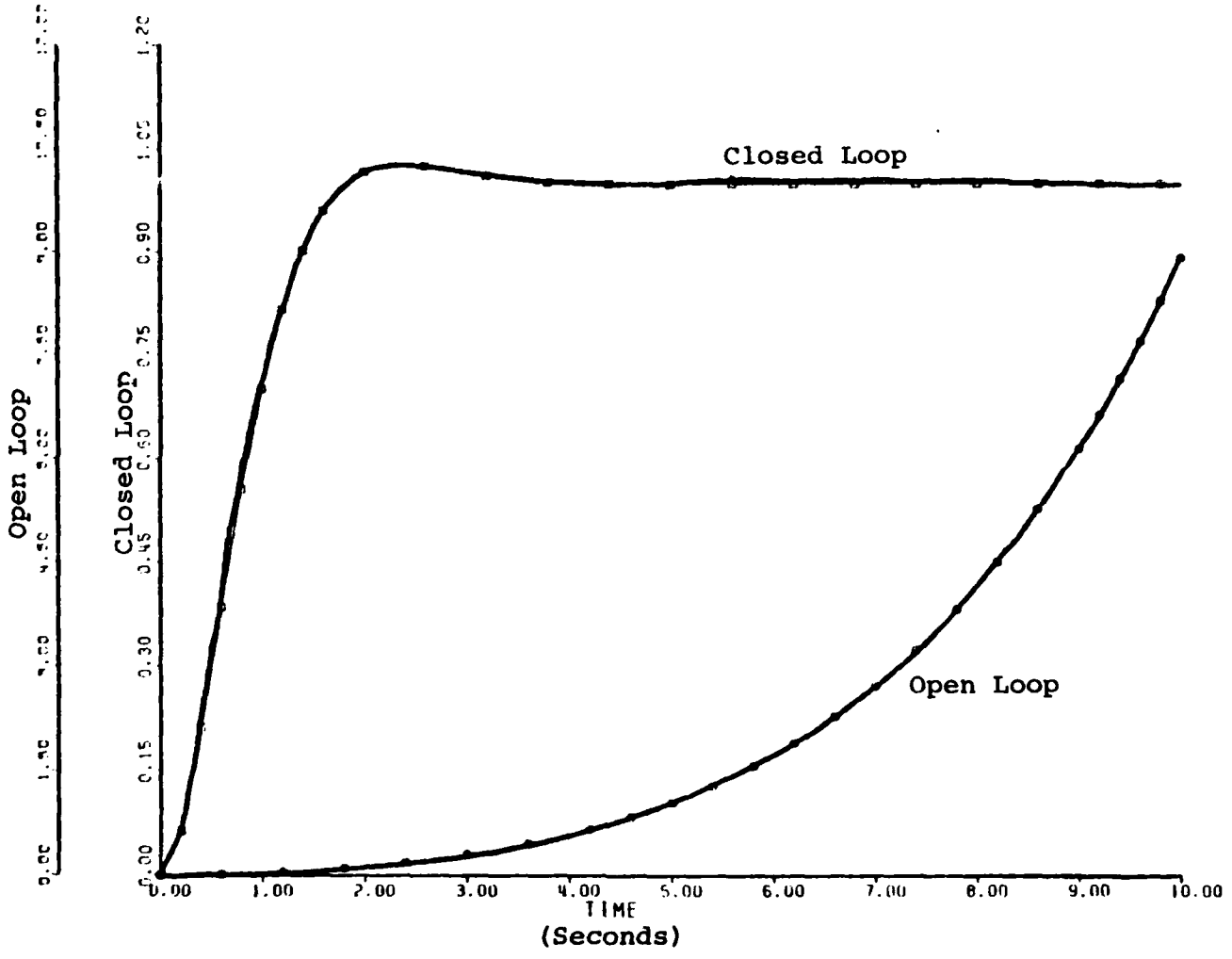

Figure 5-3 Simulation of the unit step response of the pitch attitude of the AV-8A Harrier with SRFIMF control.

in equation 5-1 in matrix notation as

$$\begin{Bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \\ \dot{X}_4 \end{Bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -.00747 & -.06736 & +.4495 & -.4896 \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{Bmatrix} + \begin{Bmatrix} 0 \\ .25 \\ -.061 \\ .1443 \end{Bmatrix} \delta_e \qquad 5\text{-}2$$

The control vector of equation 5-2 is determined by the numerator of equation 5-1 so that the states X_1 and X_2 are position and velocity. The algorithm for determining the required control vector is given by Ogata, reference [13].

The remaining state variables are defined in the following way

X_5 = output of the control actuator, input to the plant

X_6 = output of the compensator

X_7 = state variable representing velocity sensor error

X_8 = state variable representing acceleration sensor error

X_9 = state variable representing position sensor error

Figure 5-4 is the schematic representation of the system and from the figure the control law is obtained as

$$W(t) = -K_x X_1 - K_x X_2 - X_3 - CPG\ X_5 + X_6 - K_x X_7 - X_8 - K_x X_9 \qquad 5\text{-}3$$

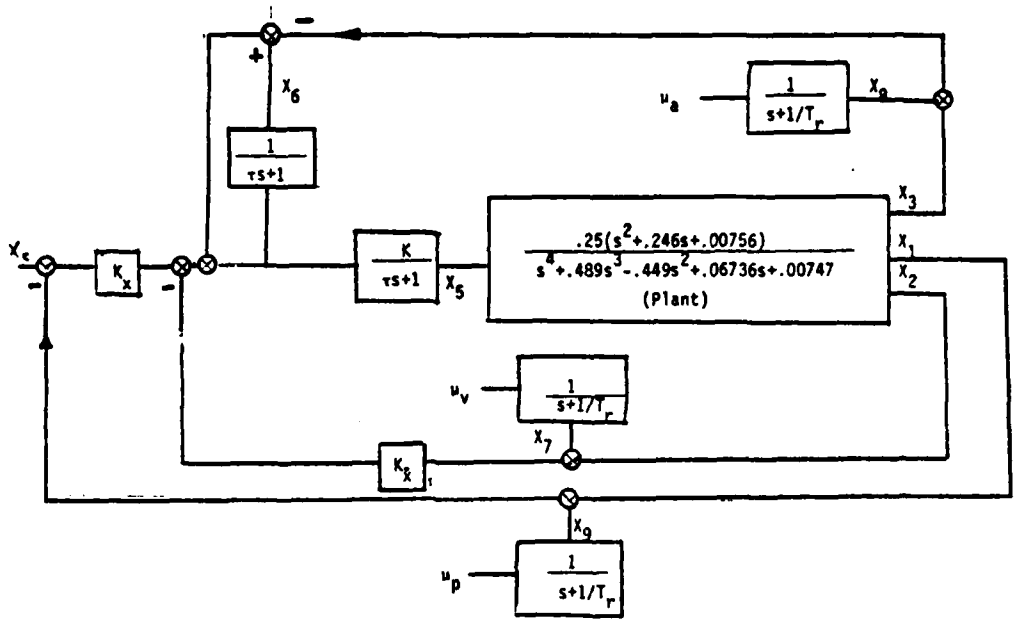

Figure 5-4 Schematic representation of the AV-8A Harrier pitch axis with the SRFIMF controller. Including sensor noise.

Combining and expressing the system in matrix form, the closed loop system is given by

$$\dot{X} = AX + BQ \qquad 5\text{-}4$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & .25 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -.06115 & 0 & 0 & 0 & 0 \\ .00748 & -.0675 & .45 & -.4906 & .1443 & 0 & 0 & 0 & 0 \\ -KK_x/\tau & -KK_{\dot{x}}/\tau & -K/\tau & 0 & -(.25K+1)/\tau & K/\tau & -KK_{\dot{x}}/\tau & -K/\tau & -KK_x/\tau \\ -K_x/\tau & -K_{\dot{x}}/\tau & -1/\tau & 0 & -.25/\tau & 0 & -K_{\dot{x}}/\tau & -1/\tau & -K_x/\tau \\ 0 & 0 & 0 & 0 & 0 & 0 & -1/T_r & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1/T_r & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1/T_r \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad Q = \begin{bmatrix} 2\sigma_v^2/T_r & 0 & 0 \\ 0 & 2\sigma_a^2/T_r & 0 \\ 0 & 0 & 2\sigma_p^2/T_r \end{bmatrix}$$

Having the system, represented in state space, the technique used in the previous section can be applied. The general solution will be obtained for a unit value of sensor error standard deviation and later applied to the specific sensor suits assumed earlier. The constants required to obtain the numerical solution are: $K = 20$; $\tau = 0.1$; $K_x = 4$; $K_{\dot{x}} = 3$; $KRL = K \cdot CPG/\tau = 50$. The resulting system is

$$\dot{X} = AX + BQ \qquad 5\text{-}5$$

$$Y = CX$$

$$\dot{X}^* = A^*X^* + B^*Q \qquad 5\text{-}6$$

$$Y = C^*X^*$$

(Matricies A, B, C, A*, B* and C* are shown on the following pages.)

The result of the modal transformation is that the noise sources have about the same controllability and observability as in the

$$A = \begin{bmatrix} 0.0 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 & 0.25 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 & -0.06 & 0.0 & 0.0 & 0.0 & 0.0 \\ -0.01 & -0.07 & 0.45 & -0.49 & 0.14 & 0.0 & 0.0 & 0.0 & 0.0 \\ -800.0 & -800.0 & -200.0 & 0.0 & -60.0 & 200.0 & -600.0 & -200.0 & -800.0 \\ -40.0 & -30.0 & -10.0 & 0.0 & -2.5 & 0.0 & -30.0 & -10.0 & -40.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -0.1 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -0.1 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -0.1 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \\ 1.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 1.0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 \\ 0.0 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 0.0 \end{bmatrix}$$

$$A^* = \begin{bmatrix} -47.17 & 0.0 & 0.01 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & -1.54 & 1.37 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & -1.37 & -1.54 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & -0.21 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & -0.03 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -10.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -0.1 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -0.1 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -0.1 \end{bmatrix}$$

$$B^* = \begin{array}{c} \begin{matrix} Q_v & Q_a & Q_p \end{matrix} \\ \begin{bmatrix} -13.55 & -4.52 & -18.07 \\ 10.18 & 3.39 & 13.57 \\ -7.15 & -2.38 & -9.53 \\ 5.41 & 1.80 & 7.21 \\ 1.25 & 0.42 & 1.67 \\ 0.0 & 0.0 & 0.0 \\ 2.26 & 0.0 & 0.0 \\ 0.0 & 1.21 & 0.0 \\ 0.0 & 0.0 & 2.88 \end{bmatrix} \end{array}$$

$$C^* = \begin{array}{c} \begin{matrix} & & & & & & x_v^* & x_a^* & x_p^* \end{matrix} \\ \begin{bmatrix} 0.0 & 0.01 & -0.1 & 0.0 & 0.0 & 0.0 & -0.36 & -0.22 & -0.03 \\ 0.01 & 0.11 & 0.16 & 0.0 & 0.0 & -0.03 & 0.48 & 0.02 & 0.04 \\ 0.0 & -0.01 & -0.1 & -0.24 & 0.24 & 0.01 & 0.19 & 0.95 & 0.20 \end{bmatrix} \end{array}$$

earlier cases. It can be seen from the A* matrix that the closed loop system has two additional eigenvalues. These correspond to the shifted poles of the system. Variance analysis was done in the same way as in the earlier examples and the results are shown plotted in figures 5-5 through 5-8 and tabulated in Appendix B, table B-4. Table 5-1 lists the steady state values of the expected error in states X_1, X_2 and X_5.

	X_1	X_2	X_5
Position measurement error	0.98	0.36	3.85
Velocity measurement error	0.73	0.27	2.90
Acceleration measurement error	0.25	0.089	0.96

Table 5-1 Steady state values of σ_1, σ_2 and σ_5 as a result of position, velocity and acceleration measurement error.

Comparison of table 5-1 with table 4-2 indicates that the values of the standard deviation of the X_1 and X_2 errors are identical in both cases even though the two plants are very different. The standard deviation of the plan input error (X_5 here, X_3 in section IV) is slightly different in the two cases. This should be expected because of the difference in the value of control gain K, required to yield a value of KRL equal to 50 and the large difference between the two plants. From the variance analysis, we obtain an estimate of the expected errors in X_1, X_2 and X_5 which result from the sensor suit assumed in section IV. These values are listed in table 5-2.

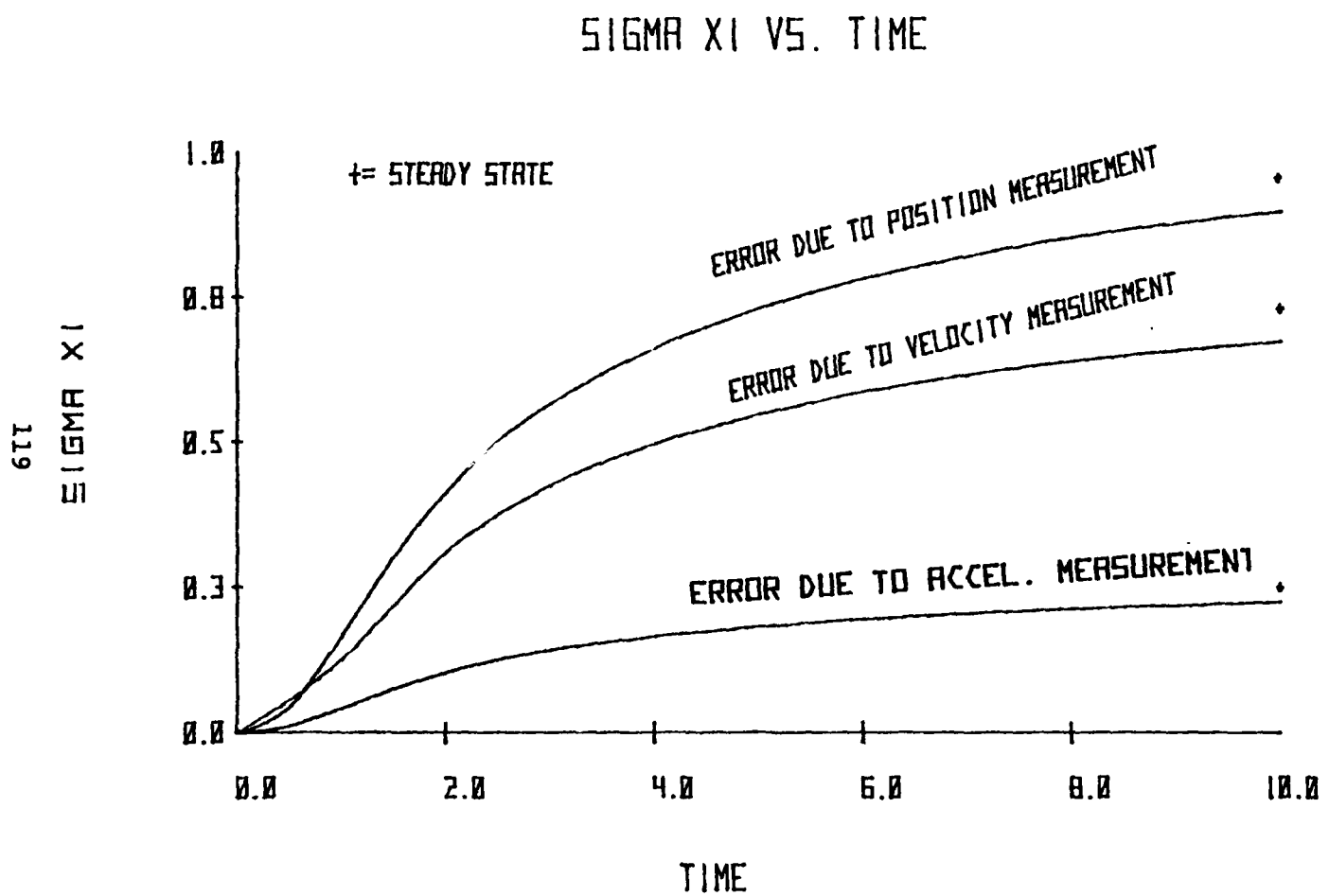

Figure 5-5 Tracking error in X_1 as a result of measurement error in position, velocity and acceleration for the Harrier longitudinal axis position control.

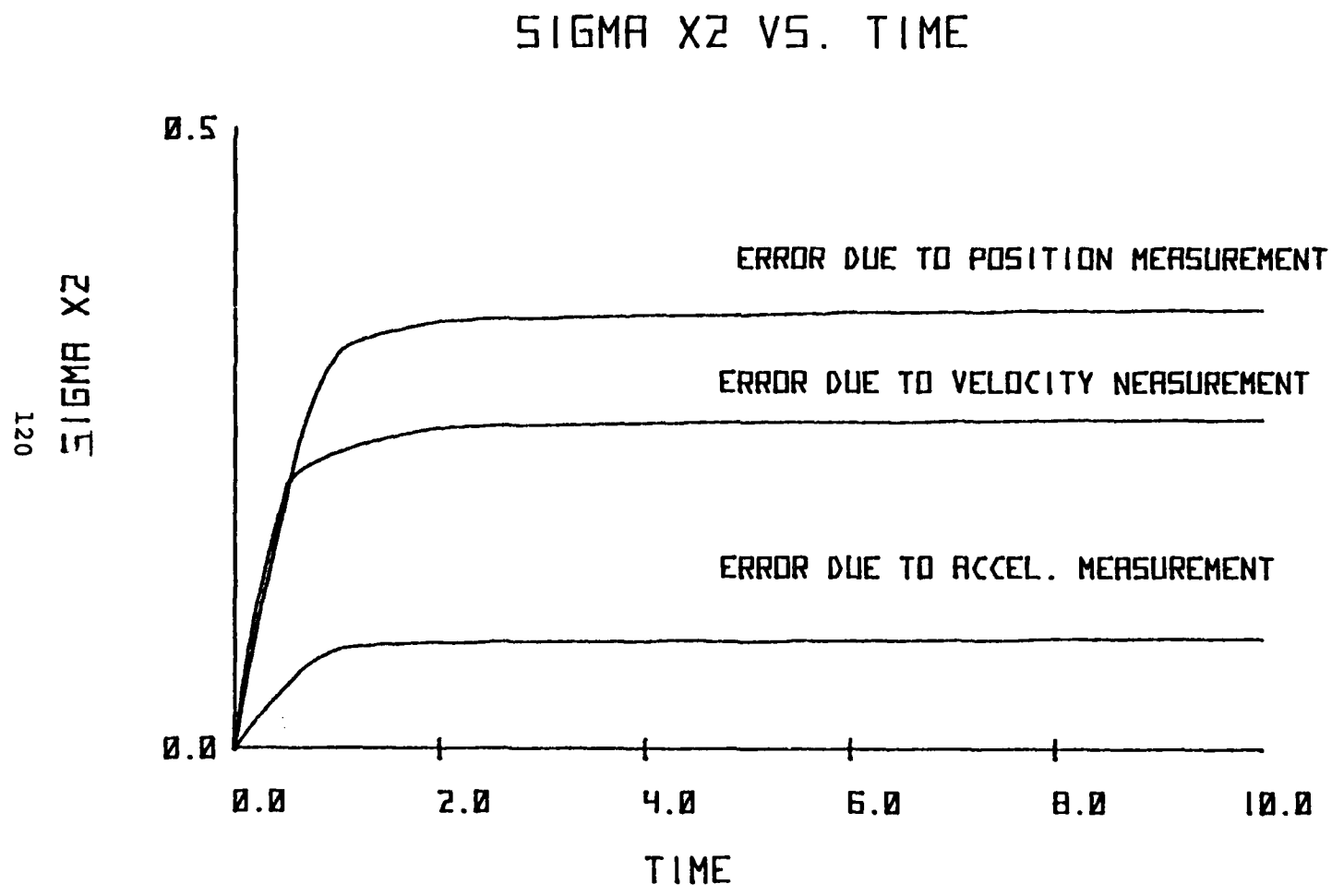

Figure 5-6 Tracking error in X_2 as a result of measurement error in position, velocity and acceleration for the Harrier longitudinal axis positon control.

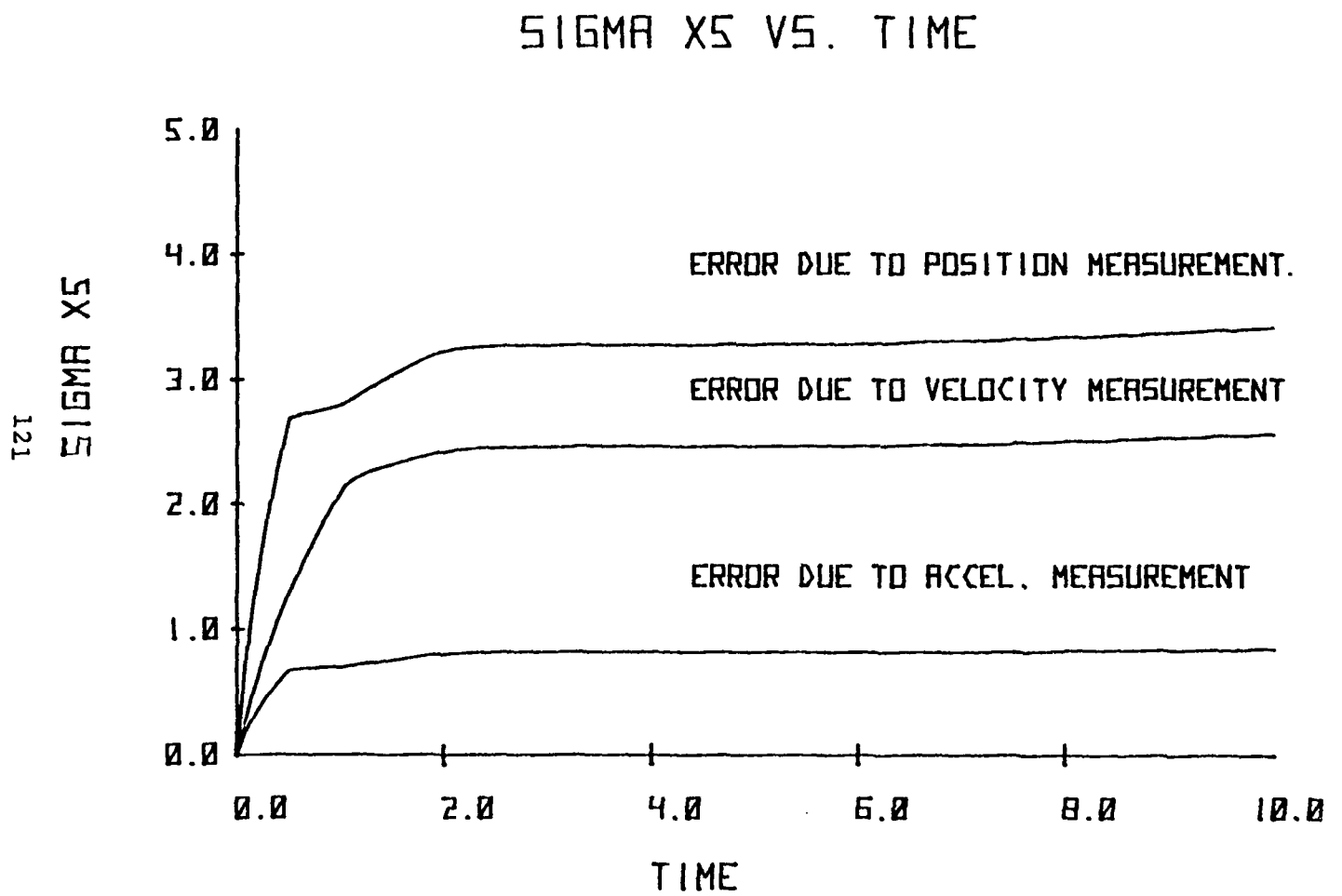

Figure 5-7 Tracking error in X_5 as a result of measurement error in position, velocity and acceleration for the Harrier longitudinal axis position control.

	X_1	X_2	X_5
Inertial sensors	0.49	0.18/sec	$1.93/sec^2$
Strapdown sensors	1.575	0.584/sec	$6.26/sec^2$

Table 5-2 Standard deviation of steady state tracking errors in X_1, X_2 and X_5 as a result of sensor noise in the Harrier aircraft.

The Significance of the total tracking error in X_5 is determined by comparing the standard deviation of the error to the value of X_5 when a 1° input is applied to the system. In quiescent operation, a 1° step input results in an instantaneous value of X_5 of

$$X_5 = .0175\text{rad} \cdot 1^\circ K_x \quad K = 1.4 \text{ rad/sec}^2$$
$$= 80^\circ/\text{sec}^2$$

The value of X_5 given in table 5-2 for strapdown sensors whose expected error is in the order of 1° is not large compared to a 1° input. We conclude that sensor measurement errors do not adversely affect the performance of the SRFIMF Harrier pitch controller.

B. PITCH ATTITUDE GUST RESPONSE

The pitch attitude response of the Harrier from a gust input will be considered by applying the same type of analysis used earlier. For this purpose assume that the gust acts as an additional, uncontrolled input to the system. The symbol δ_g is used to denote this input. The gust input will be

modeled as an exponentially correlated noise source in the same way that the sensor noise was modeled. The schematic representation of an external exponentially correlated noise source was shown in figure 4-1. In this case the output of the noise source, $\xi(s)$, is the gust input to the system and $W_t(s)$ is the white noise input of strength μ_g as given by equation 4-12. The standard deviation and correlation time for the gust can be determined by examining the Dryden wind model used in simulation by NASA. At altitudes of 500 ft. and above the Dryden model assumes that the RMS value of atmospheric turbulence, σ_g, is given as

$$\sigma_g = 0.2\ V_{wind}$$

The corrleation time, T_g, is determined by a characteristic length, L_w, divided by the vehicle speed, V, or

$$T_g = L_w/V$$

The Dryden model gives the characteristic length of the turbulence as

$$150 \text{ ft.} + h = L_w$$

where h is the altitude. Choosing 500 ft. as the flight altitude, a flight speed of 60 kts and 15 kts as the value of the wind, the standard deviation and correlation time is

$$\sigma_g = 5 \text{ ft/sec}, \quad T_g = 6.5 \text{ sec}$$

The gust input is represented schematically by figure 5-8.

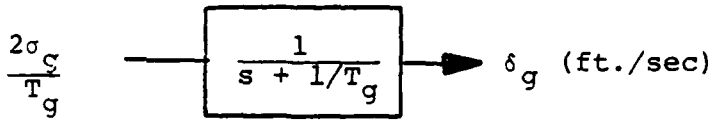

Figure 5-8 Gust shaping filter.

The transfer function between the gust and position, velocity and acceleration of the vehicle can be determined from the stability derivatives given by reference [14]. For This example we chose pitch attitude position control as was done in the previous example therefore the denominator of the gust transfer function is the same as the denominator of equation 5-1. The numerator is given by McRuer [14] for a vertical gust as

$$\frac{U_o N_g^w}{s} = (M_q - M_\alpha)s^2 + (M_\alpha - Z_w (M_q - M_\alpha)s + X_u (M_\alpha + Z_w M_q) - X_w (V \cdot M_u + Z_u M_q) \qquad 5\text{-}5$$

The stability derivatives for the Harrier at 60 kts are (6)

$M_q = -2.6$

$M_\alpha = V \cdot M_w = .54$

$Z_w = -.190$

$Z_u = -.036$

$X_u = -0.43$

$X_w = -0.27$

$M_u = .022$

From equation 5-5 and 5-1, we obtain the transfer function for the pitch attitude from a gust input

$$\frac{\theta(s)}{\delta_g(s)} = \frac{-.0026s\,[s^2 + 2.31s + .073]}{s^4 + .4896s^3 - .4495s^2 + .0636s + .00747} \qquad 5\text{-}6$$

The transfer function from the pilot's stick to the pitch angle is given by equation 5-1 where the input to the plant is δ_e. Expressing the plant, with multiple inputs of stick and gust load, in matrix notation with states X_1 and X_2 defined as position and velocity we have

$$\begin{Bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \\ \dot{X}_4 \end{Bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -.00747 & -.06736 & +.4495 & -.4896 \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{Bmatrix} + \begin{bmatrix} 0 & -.0026 \\ .25 & -.00473 \\ -.061 & .00306 \\ .1443 & .00345 \end{bmatrix} [\delta_e \ \delta_g]$$

The B matrix has been determined as before so that states X_1 and X_2 represent position and velocity.

Having the plant transfer function defined by 5-7 we can develope the equation to represent the closed loop controller. The additional state variables required are defined

X_5 = output of the control actuator, input to plant

X_6 = output of the compensator

X_7 = gust input to the plant, $\delta_g(s)$, figure 5-5.

The schematic of the closed loop system is shown in figure 5-9.

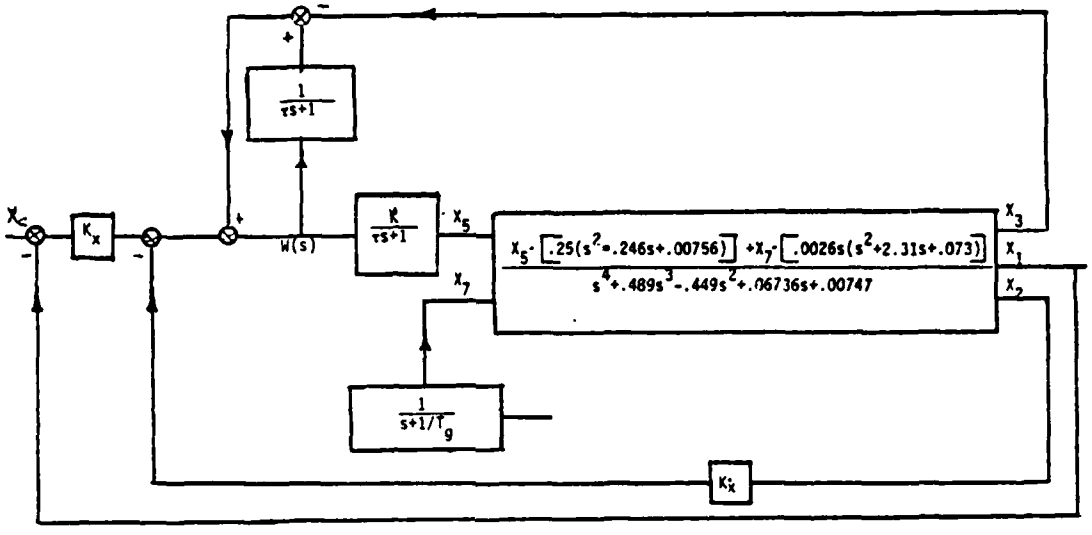

Figure 5-9 Schematic representation of the AV-8A Pitch control using a SRFIMF controller with wind gust input.

With $X_c = 0$, the control law W(t) is

$$W(t) = -K_x X_1 - K_{\dot{x}} X_2 - X_3 - .25X_5 + X_6 + (K_{\dot{x}} \cdot .0026 + .00473)X_7$$

Rearranging in a manner similar to earlier work, we write the closed loop equation in matrix notation as

$$\dot{X} = AX + BQ \qquad 5\text{-}8$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & -.0026 \\ 0 & 0 & 1 & 0 & .25 & 0 & -.00473 \\ 0 & 0 & 0 & 1 & -.061 & 0 & .00306 \\ -.00748 & -.0675 & .45 & -.49 & .1443 & 0 & .00346 \\ -K \cdot K_x/\tau & -KK_{\dot{x}}/\tau & -K/\tau & 0 & -(.25 \cdot K+1)/\tau & K/\tau & (K_{\dot{x}} \cdot .0026+.00473)K/\tau \\ -K_x/\tau & -K_{\dot{x}}/\tau & -1/\tau & 0 & -.25/\tau & -K_{\dot{x}}/\tau & (K_{\dot{x}} \cdot .0026+.00473)/\tau \\ 0 & 0 & 0 & 0 & 0 & 0 & -1/T_g \end{bmatrix}$$

$$B = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{Bmatrix} \qquad Q = \left[2\sigma_g / T_g \right]$$

The covariance of the states were calculated for the system described above as was done in previous examples. The results of the covariance analysis was that the RMS values of X_1 and X_2 are very small and quickly reach the steady state values of

$$\sigma_p = \sigma_{X_1} = .0017 \qquad \sigma_v = \sigma_{X_2} = .0134$$

In the case of the gust we are interested in the pitch acceleration of the vehicle as a result of the gust input. Acceleration is given as $\dot{X}_2$ and from equation 5-8 is

$$\text{Acceleration} = X_3 + .25X_5 - .00473X_7 \qquad 5\text{-}9$$

Recalling that the elements of the covariance matrix, P, are the squares of the standard deviations of the state variables, we write from equation 5-9 that

$$\sigma_{accel} = P_{33}^{\frac{1}{2}} + .25P_{55}^{\frac{1}{2}} - .00473P_{77}^{\frac{1}{2}}$$

The standard deviation in pitch acceleration as a result of a gust input was plotted along with the gust input and the RMS value of X_5 as a function of time in figure 5-7. The results are also listed in Appendix B table B-5. The gust is shown for comparison of response time.

It is seen from the figure that the rise time in the RMS value of the vehicle acceleration is much slower than the gust itself. This result can be interpretated as a smoothing of the gust by the aircraft. This is primarily due to the slow response of the aircraft to the gust. The controller has a relatively fast reaction time and can maintain the output X_1 with only very small errors as a result of a gust input.

The analysis of the SRFIMF controller applied to the Harrier has indicated that the qualities of the controller found by analysis when the plant is assumed to be a second order system apply equally well when the plant is an actual VSTOL aircraft. We have also seen that the controller does not produce undesireable dynamic response when the vehicle is subjected to gust inputs and that the dynamic response of the aircraft and controller is due to the response of the aircraft alone to the gust.

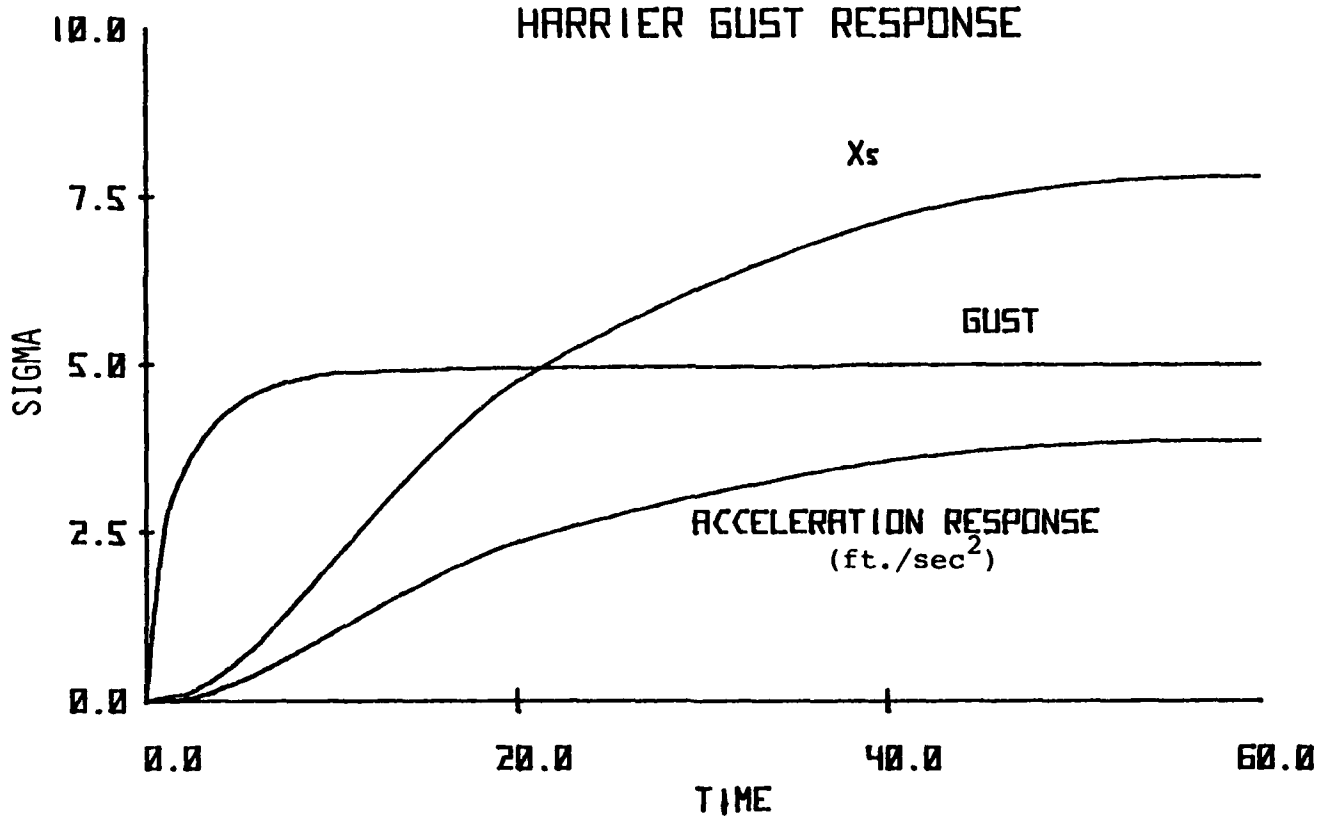

Figure 5-10 Standard deviation of vehicle pitch acceleration and plant input, X_5, as a result of a gust.

VI. CONCLUSIONS

The use of State Rate Feedback Implicit Model Following, for the attitude control of VSTOL aircraft, has been studied. The SRFIMF control scheme is simple, easy to implement and can be expected to be reliable. It was shown that the dynamic response of the closed loop system is a second order response and that the natural frequency and damping are free choices of the designer. It was also shown that sensor noise does not adversely affect the operation of the system. Detailed conclusions of this study are:

1. Model following was shown in section III-D by simulations to be very good. Despite changing plant dynamics, representing a variety of flight conditions, the closed loop system had a response characteristic of that of the model and that the output of the closed loop system was driven to the value of the input.

2. The frequency response of the system is that of a low pass filter and that there was no phase shift between the output and the input at low frequencies.

3. Non-minimum phase system, pole-zero cancellation can lead to unstable performance.

4. High quality sensors are not required. It was shown by the covariance analysis in section IV-B that the error in position is approximately equal to the error in position

measurement plus 75 percent of the error in velocity measurement plus 25 percent of the error in acceleration measurement. The general result is that sensor errors do not adversely affect the performance of the system.

5. Estimation of position and velocity by integration of acceleration was shown to lead to unstable performance because of neutrally stable modes resulting from the integration. It was shown in section IV-C that acceleration sensor noise disturbs the neutrally stable modes with the result that the system diverges. It is then concluded that the quantities of position and velocity must be measured in order to have a stable system which does not require knowledge of the plant.

6. The system is capable of compensating for gust inputs. The analysis of section V-C showed that the error in position output was small as a result of the gust. Secondly, the response of the system to the gust is smoothed by the action of the uncontrolled plant.

APPENDIX A

COMPUTER LISTINGS

```
************************************************************************
*
*         ***  SRFIMF SIMULATION SOURCE CODE  ***
*
*      CSMP SOURCE CODE USED TO SIMULATE THE RESPONSE OF
*      THE SRFIMF AND AN ARBITRARY SECOND ORDER PLANT.
*      THE FUNCTIONS INTGRL AND REALPL ARE CSMP FUNCTIONS
*      WHICH PREFORM INTEGRATION OR SIMULATE A FIRST ORDER
*      RESPONSE.THE ORDER OF THE STATEMENTS IS UNIMPORTANT IN
*      CSMP.
*
************************************************************************
// EXEC CSMPXV
//X.COMPRINT DD DUMMY
//X.SYSPRINT DD DUMMY
//X.SYSIN DD *
       AS=REALPL(0,TOU,WS)
       X=INTGRL(0,VX)
       VX=INTGRL(0,AX)
       AX1=REALPL(0,TOU,IS)
       AX=AX1*CPG-C*X-B*VX
       IS=WS*K
       WS=-KXD*VX+(XC-X)*KX+VS
       VS=AS-AX
*
*      SIMULATION OF THE OPEN LOOP PLANT
*
       XDD=CPG*AX2-C*XP-VXP*B
       AX2=REALPL(0.,TOU,XC)
       VXP=INTGRL(0,XDD)
       XP=INTGRL(0,VXP)
       XC=G*STEP(0)
       G=1.
       CPG=4.
       TOU=.1
       K=5.
       KXD=3.
       KX=4.
       B=.6
       C=2.16
TIMER  FINTIM=10,OUTDEL=.2
OUTPUT X,XP
OUTPUT TIME,XP
PAGE   XYPLOT
       END
       STOP
```

```
C*************************************************************************      MOD00040
C                                                                               MOD00050
C                                                                               MOD00060
C                                    MAIN 1                                     MOD00070
C                                                                               MOD00080
C     MAIN 1 IS THE CODE USED TO GENERATE THE A,B, AND C MATRICIES FOR          MOD00090
C     THE BASIC STOCHASTIC SRFIMF.POSITION,VELOCITY AND ACCELERATION            MOD00100
C     ARE ASSUMED TO BE MEASURED AND THE PLANT IS A SECOND ORDER                MOD00110
C     SYSTEM.THE PROGRAM CALLS VARY TO DETERMINE THE SOLUTION TO                MOD00120
C     THE LYAPUNOV EQUATION AND MODAL FOR THE MODAL ANALYSIS.                   MOD00130
C                                                                               MOD00140
C                                                                               MOD00150
C*************************************************************************      MOD00160
      DIMENSION B(8,4),TR(4),H(3,8)                                             MOD00170
      REAL K,KX,KXD                                                             MOD00180
      COMMON BP,CP,K,KX,KXD,TOU,CPG,N,PD(64),Q(8,8),A(8,8)                      MOD00190
      WRITE(6,100)                                                              MOD00200
100   FORMAT(//,20X,90('*'),//,40X,'MODEL 1   POSITION,VELOCITY AND ',          MOD00210
     1'ACCELERATION MEASURED',//,20X,90('*'))                                   MOD00220
      N=8                                                                       MOD00230
      NCOUNT=0                                                                  MOD00240
      M=3                                                                       MOD00250
      L=4                                                                       MOD00260
      BP=.6                                                                     MOD00270
      CP=2.15                                                                   MOD00280
      K=5.                                                                      MOD00290
      KXD=3.                                                                    MOD00300
      KX=4.                                                                     MOD00310
      TR(1)=10.                                                                 MOD00320
      TR(2)=10.                                                                 MOD00330
      TR(3)=10.                                                                 MOD00340
      TR(4)=10.                                                                 MOD00350
      SIGMA1=1.                                                                 MOD00360
      TOU=.1                                                                    MOD00370
      CPG=1.                                                                    MOD00380
      SIGMA2=1.                                                                 MOD00390
      SIGMA3=1.                                                                 MOD00400
C                                                                               MOD00410
CC    COMPUTE A,B,C                                                             MOD00420
C                                                                               MOD00430
      DO 9 I=1,N                                                                MOD00440
      DO 9 J=1,N                                                                MOD00450
9     A(I,J)=0.0                                                                MOD00460
      DO 5 I=1,N                                                                MOD00470
      DO 5 J=1,L                                                                MOD00480
5     B(I,J)=0.0                                                                MOD00490
      A(1,2)=1.                                                                 MOD00500
      A(2,1)=-CP                                                                MOD00510
```

```
      A(2,2)=-BP                                                        MOD00520
      A(2,3)=CPG                                                        MOD00530
      A(3,1)=(CP-KX)*K/TOU                                              MOD00540
      A(3,2)=(BP-KXD)*K/TOU                                             MOD00550
      A(3,3)=-(CPG*K+1.)/TOU                                            MOD00560
      A(3,4)=K/TOU                                                      MOD00570
      A(3,5)=-KXD*K/TOU                                                 MOD00580
      A(3,6)=-K/TOU                                                     MOD00590
      A(3,7)=-K*KX/TOU                                                  MOD00600
      A(3,8)=-A(3,7)                                                    MOD00610
      A(4,1)=A(3,1)/K                                                   MOD00620
      A(4,2)=A(3,2)/K                                                   MOD00630
      A(4,3)=-CPG/TOU                                                   MOD00640
      A(4,5)=-KXD/TOU                                                   MOD00650
      A(4,6)=-1./TOU                                                    MOD00660
      A(4,7)=-KX/TOU                                                    MOD00670
      A(4,8)=KX/TOU                                                     MOD00680
      A(5,5)=-1./TR(1)                                                  MOD00690
      A(6,6)=-1./TR(2)                                                  MOD00700
      A(7,7)=-1./TR(3)                                                  MOD00710
      A(8,8)=-1./TR(4)                                                  MOD00720
C                                                                       MOD00730
      B(5,1)=1.                                                         MOD00740
      B(6,2)=1.                                                         MOD00750
      B(7,3)=1.                                                         MOD00760
C                                                                       MOD00770
      DO 1 I=1,M                                                        MOD00780
      DO 1 J=1,N                                                        MOD00790
    1 H(I,J)=0.0                                                        MOD00800
      H(1,1)=1.                                                         MOD00810
      H(1,7)=1.                                                         MOD00820
      H(2,2)=1.                                                         MOD00830
      H(2,5)=1.                                                         MOD00840
      H(3,1)=-CP                                                        MOD00850
      H(3,2)=-BP                                                        MOD00860
      H(3,3)=CPG                                                        MOD00870
      H(3,6)=1.                                                         MOD00880
C                                                                       MOD00890
      DO 4 I=1,L                                                        MOD00900
      DO 4 J=1,L                                                        MOD00910
    4 Q(I,J)=0.0                                                        MOD00920
   20 Q(1,1)=(2./TR(1))*SIGMA1**2                                       MOD00930
      Q(2,2)=(2./TR(2))*SIGMA2**2                                       MOD00940
      Q(3,3)=(2./TR(3))*SIGMA3**2                                       MOD00950
      Q(4,4)=(2./TR(4))*SIGMA4**2                                       MOD00960
C                                                                       MOD00970
      NPT=3                                                             MOD00980
      T1=0.0                                                            MOD00990
```

```
      T2=60.                                                            MOD01000
      CALL ECHO(A,B,Q,H,TR,N,M,L)                                       MOD01010
      CALL VARY(B,L,NPT,T1,T2)                                          MOD01020
      CALL MODAL(A,B,H,N,M,L)                                           MOD01030
      STOP                                                              MOD01040
      END                                                               MOD01050
C*********************************************************************  MOD01060
CCC                                                                     MOD01070
CCC                            MAIN 2                                   MOD01080
CCC                                                                     MOD01090
CCC   MAIN 2 IS THE CODE USED TO GENERATE THE A,B  AND C                MOD01100
CCC   MATRICIES FOR THE FIRST ALTERNATE MEASUREMENT SCHEME.             MOD01110
CCC   IN THIS CASE ACCELERATION IS MEASURED,POSITION                    MOD01120
CCC   AND VELOCITY ARE ESTIMATED.                                       MOD01130
CCC                                                                     MOD01140
C*********************************************************************  MOD01150
      DIMENSION B(8,4),H(3,8),TR(2)                                     MOD01160
      REAL K,KX,KXD                                                     MOD01170
      COMMON BP,CP,K,KX,KXD,TOU,CPG,N,PD(64),Q(8,8),A(8,8)              MOD01180
      WRITE(6,100)                                                      MOD01190
100   FORMAT(20X,90('*'),//,40X,'MODEL 2  MEASURED ACCELERATION',       MOD01200
     1' IMPLIED POSITION AND VELOCITY',//,20X,90('*'))                  MOD01210
      N=8                                                               MOD01220
      M=3                                                               MOD01230
      L=2                                                               MOD01240
      NCOUNT=0                                                          MOD01250
C                                                                       MOD01260
CCC   INITIALIZE MATRICIES TO ZERO                                      MOD01270
CC                                                                      MOD01280
      DO 1 I=1,N                                                        MOD01290
      DO 1 J=1,N                                                        MOD01300
1     A(I,J)=0.0                                                        MOD01310
      DO 2 I=1,N                                                        MOD01320
      DO 2 J=1,L                                                        MOD01330
2     B(I,J)=0.0                                                        MOD01340
      DO 3 I=1,L                                                        MOD01350
      DO 3 J=1,L                                                        MOD01360
3     Q(I,J)=0.0                                                        MOD01370
      DO 4 I=1,M                                                        MOD01380
      DO 4 J=1,N                                                        MOD01390
4     H(I,J)=0.0                                                        MOD01400
CC                                                                      MOD01410
CCC   CALCULATE THE A,B,C MATRICIES                                     MOD01420
C                                                                       MOD01430
      A(1,2)=1.                                                         MOD01440
      A(2,1)=-CP                                                        MOD01450
      A(2,2)=-BP                                                        MOD01460
      A(2,3)=CPG                                                        MOD01470
```

```
      A(3,1)=CP*K/TOU                                                   MOD01480
      A(3,2)=BP*K/TOU                                                   MOD01490
      A(3,3)=-(CPG*K+1.)/TOU                                            MOD01500
      A(3,4)=K/TOU                                                      MOD01510
      A(3,5)=-K*KXD/TOU                                                 MOD01520
      A(3,6)=-K*KX/TOU                                                  MOD01530
      A(3,7)=-K/TOU                                                     MOD01540
      A(3,8)=K*KX/TOU                                                   MOD01550
      A(4,1)=CP/TOU                                                     MOD01560
      A(4,2)=BP/TOU                                                     MOD01570
      A(4,3)=-CPG/TOU                                                   MOD01580
      A(4,5)=-KXD/TOU                                                   MOD01590
      A(4,6)=-KX/TOU                                                    MOD01600
      A(4,7)=-1./TOU                                                    MOD01610
      A(4,8)=KX/TOU                                                     MOD01620
      A(5,1)=-CP                                                        MOD01630
      A(5,2)=-BP                                                        MOD01640
      A(5,3)=CPG                                                        MOD01650
      A(5,7)=1.                                                         MOD01660
      A(6,5)=1.                                                         MOD01670
      A(7,7)=-1./TR(1)                                                  MOD01680
      A(8,8)=-1./TR(2)                                                  MOD01690
C                                                                       MOD01700
      B(7,1)=1.                                                         MOD01710
      B(8,2)=0.                                                         MOD01720
      H(1,1)=1.                                                         MOD01730
      H(2,2)=1.                                                         MOD01740
      H(3,1)=-CP                                                        MOD01750
      H(3,2)=-BP                                                        MOD01760
      H(3,3)=CPG                                                        MOD01770
      SIGMA1=1.                                                         MOD01780
      SIGMA2=1.                                                         MOD01790
      H(3,7)=1.                                                         MOD01800
C                                                                       MOD01810
      Q(1,1)=(2./TR(1))*SIGMA1**2                                       MOD01820
      Q(2,2)=(2./TR(2))*SIGMA2**2                                       MOD01830
      NPT=1                                                             MOD01840
      T1=0.0                                                            MOD01850
      T2=.5                                                             MOD01860
      CALL ECHO(A,B,Q,H,TR,N,M,L)                                       MOD01870
      CALL VARY(B,L,NPT,T1,T2)                                          MOD01880
      CALL MODAL(A,B,H,N,M,L)                                           MOD01890
1151  STOP                                                              MOD01900
      END                                                               MOD01910
C*********************************************************************  MOD01920
CC                                                                      MOD01930
CC                          MAIN 3                                      MOD01940
C     MAIN 3 IS THE CODE USED TO GENERATE THE A,B AND C MATRICIES       MOD01950
```

```
C     FOR THE SECOND ALTERNATE MEASUREMENT SCHEME.IN THIS                    MOD01960
CC    CASE POSITION AND VELOCITY ARE MEASURED AND ACCELERATION IS            MOD01970
C     ESTIMATED.                                                             MOD01980
C                                                                            MOD01990
C*************************************************************************** MOD02000
      DIMENSION B(7,3),TR(3),H(3,7)                                          MOD02010
      REAL K,KX,KXD                                                          MOD02020
      COMMON BP,CP,K,KX,KXD,TOU,CPG,N,PD(49),Q(7,7),A(7,7)                   MOD02030
      WRITE(6,100)                                                           MOD02040
100   FORMAT(20X,90('*'),//,40X,'MODEL 3 MEASURED POSITION AND VELOCI'       MOD02050
     1,'TY IMPLIED ACCELERATION',//,20X,90('*'))                             MOD02060
      N=7                                                                    MOD02070
      M=3                                                                    MOD02080
      L=3                                                                    MOD02090
      NCOUNT=0                                                               MOD02100
      WRITE(6,502)                                                           MOD02110
502   FORMAT(//,10X,' ERROR DUE TO POSITION MEASUREMENT',//)                 MOD02120
      SIGMA1=1.                                                              MOD02130
      SIGMA2=1.                                                              MOD02140
      SIGMA3=1.                                                              MOD02150
C                                                                            MOD02160
CC    COMPUTE A,B,C,Q                                                        MOD02170
C                                                                            MOD02180
CC    INITIALIZE ALL MATRICIES                                               MOD02190
C                                                                            MOD02200
      DO 1 I=1,N                                                             MOD02210
      DO 1 J=1,N                                                             MOD02220
1     A(I,J)=0.0                                                             MOD02230
      DO 2 I=1,N                                                             MOD02240
      DO 2 J=1,L                                                             MOD02250
2     B(I,J)=0.0                                                             MOD02260
      DO 3 I=1,L                                                             MOD02270
      DO 3 J=1,L                                                             MOD02280
3     Q(I,J)=0.0                                                             MOD02290
      DO 4 I=1,M                                                             MOD02300
      DO 4 J=1,N                                                             MOD02310
4     H(I,J)=0.0                                                             MOD02320
C                                                                            MOD02330
      A(1,2)=1.                                                              MOD02340
      A(2,1)=-CP                                                             MOD02350
      A(2,2)=-BP                                                             MOD02360
      A(2,3)=CPG                                                             MOD02370
      A(3,1)=K*(CP-KX)/TOU                                                   MOD02380
      A(3,2)=K*(BP-KXD)/TOU                                                  MOD02390
      A(3,3)=-(K*CPG+1.)/TOU                                                 MOD02400
      A(3,4)=K/TOU                                                           MOD02410
      A(3,5)=A(3,1)                                                          MOD02420
      A(3,6)=A(3,2)                                                          MOD02430
```

```
      A(3,7)=K*KX/TOU                                                   MOD02440
      A(4,1)=(CP-KX)/TOU                                                MOD02450
      A(4,2)=(BP-KXD)/TOU                                               MOD02460
      A(4,3)=-CPG/TOU                                                   MOD02470
      A(4,5)=A(4,1)                                                     MOD02480
      A(4,6)=A(4,2)                                                     MOD02490
      A(4,7)=KX/TOU                                                     MOD02500
      A(5,5)=-1./TR(1)                                                  MOD02510
      A(6,6)=-1./TR(2)                                                  MOD02520
      A(7,7)=-1./TR(3)                                                  MOD02530
      B(5,1)=1.                                                         MOD02540
      B(6,2)=0.                                                         MOD02550
      B(7,3)=0.                                                         MOD02560
      H(1,1)=1.                                                         MOD02570
      H(2,2)=1.                                                         MOD02580
      H(3,1)=-CP                                                        MOD02590
      H(3,2)=-BP                                                        MOD02600
      H(3,3)=CPG                                                        MOD02610
C                                                                       MOD02620
      Q(1,1)=(2./TR(1))*SIGMA1**2                                       MOD02630
      Q(2,2)=(2./TR(2))*SIGMA2**2                                       MOD02640
      Q(3,3)=(2./TR(3))*SIGMA3**2                                       MOD02650
      NPT=40                                                            MOD02660
      T1=0.                                                             MOD02670
      T2=20.                                                            MOD02680
      CALL ECHO(A,B,Q,H,TR,N,M,L)                                       MOD02690
      CALL VARY (B,L,NPT,T1,T2)                                         MOD02700
      CALL MODAL (A,B,H,N,M,L)                                          MOD02710
      STOP                                                              MOD02720
      END                                                               MOD02730
C*********************************************************************  MOD02740
C                                                                       MOD02750
C                          HARRIER 1                                    MOD02760
C     HARRIER 1 IS THE CODE USED TO GENERATE THE A,B AND C MATRICIES    MOD02770
C     FOR THE COVARIANCE CALCULATIONS OF THE SRFIMF CONTROLLER          MOD02780
C     AND THE LONGITUDNAL MODE OF THE HARRIER.                          MOD02790
C                                                                       MOD02800
C*********************************************************************  MOD02810
      DIMENSION B(9,3),H(3,9),TR(3)                                     MOD02820
      REAL K,KX,KXD                                                     MOD02830
      COMMON BP,CP,K,KX,KXD,TOU,CPG,N,PD(81),Q(9,9),A(9,9)              MOD02840
      WRITE(6,100)                                                      MOD02850
100   FORMAT(//,20X,90('*'),//,40X,'HARRIER MODEL ',                    MOD02860
     1' ALL QUANTITIES MEASURED',//,20X,90('*'),//)                     MOD02870
      N=9                                                               MOD02880
      L=3                                                               MOD02890
      M=3                                                               MOD02900
      K=20.                                                             MOD02910
```

```
      KX=4.                                                          MOD02920
      KXD=3.                                                         MOD02930
      TOU=.1                                                         MOD02940
      SIGMA1=1.                                                      MOD02950
      SIGMA2=1.                                                      MOD02960
      SIGMA3=1.                                                      MOD02970
      TR(1)=10.                                                      MOD02980
      TR(2)=10.                                                      MOD02990
      TR(3)=10.                                                      MOD03000
C                                                                    MOD03010
CC    CALCULATE THE A MATRIX                                         MOD03020
C                                                                    MOD03030
      DO 1 I=1,N                                                     MOD03040
      DO 1 J=1,N                                                     MOD03050
    1 A(I,J)=0.0                                                     MOD03060
      A(1,2)=1.                                                      MOD03070
      A(2,3)=1.                                                      MOD03080
      A(2,5)=.25                                                     MOD03090
      A(3,4)=1.                                                      MOD03100
      A(3,5)=-.06115                                                 MOD03110
      A(4,1)=-.00748                                                 MOD03120
      A(4,2)=-.0675                                                  MOD03130
      A(4,3)=.45                                                     MOD03140
      A(4,4)=-.4906                                                  MOD03150
      A(4,5)=.14428                                                  MOD03160
      A(5,1)=-K*KX/TOU                                               MOD03170
      A(5,2)=-K*KXD/TOU                                              MOD03180
      A(5,3)=-K/TOU                                                  MOD03190
      A(5,5)=-(.25*K+1.)/TOU                                         MOD03200
      A(5,6)=K/TOU                                                   MOD03210
      A(5,7)=-K*KXD/TOU                                              MOD03220
      A(5,8)=-K/TOU                                                  MOD03230
      A(5,9)=-K*KX/TOU                                               MOD03240
      A(6,1)=-KX/TOU                                                 MOD03250
      A(6,2)=-KXD/TOU                                                MOD03260
      A(6,3)=-1./TOU                                                 MOD03270
      A(6,5)=-.25/TOU                                                MOD03280
      A(6,7)=-KXD/TOU                                                MOD03290
      A(6,8)=-1./TOU                                                 MOD03300
      A(6,9)=-KX/TOU                                                 MOD03310
      A(7,7)=-1./TR(1)                                               MOD03320
      A(8,8)=-1./TR(2)                                               MOD03330
      A(9,9)=-1./TR(3)                                               MOD03340
C                                                                    MOD03350
CC    B MATRIX                                                       MOD03360
C                                                                    MOD03370
      DO 2 I=1,N                                                     MOD03380
      DO 2 J=1,L                                                     MOD03390
```

```
2       B(I,J)=0.0                                                      MOD03400
        B(7,1)=1.                                                       MOD03410
        B(8,2)=1.                                                       MOD03420
        B(9,3)=1.                                                       MOD03430
20      CONTINUE                                                        MOD03440
C                                                                       MOD03450
CC      Q MATRIX                                                        MOD03460
C                                                                       MOD03470
        DO 3 I=1,M                                                      MOD03480
        DO 3 J=1,M                                                      MOD03490
3       Q(I,J)=0.0                                                      MOD03500
        Q(1,1)=(2./TR(1))*SIGMA1**2                                     MOD03510
        Q(2,2)=(2./TR(2))*SIGMA2**2                                     MOD03520
        Q(3,3)=(2./TR(3))*SIGMA3**2                                     MOD03530
C                                                                       MOD03540
CC      C MATRIX                                                        MOD03550
C                                                                       MOD03560
        DO 4 I=1,M                                                      MOD03570
        DO 4 J=1,N                                                      MOD03580
4       H(I,J)=0.0                                                      MOD03590
        H(1,1)=1.0                                                      MOD03600
        H(1,9)=1.                                                       MOD03610
        H(2,2)=1.0                                                      MOD03620
        H(2,7)=1.0                                                      MOD03630
        H(3,3)=1.0                                                      MOD03640
        H(3,8)=1.                                                       MOD03650
        NPT=40                                                          MOD03660
        T1=0.                                                           MOD03670
        T2=20.                                                          MOD03680
        CALL ECHO(A,B,Q,H,TR,N,M,L)                                     MOD03690
        CALL MODAL(A,B,H,N,M,L)                                         MOD03700
        CALL VARY (B,L,NPT,T1,T2)                                       MOD03710
        STOP                                                            MOD03720
        END                                                             MOD03730
C*****************************************************************     MOD03740
C                                                                       MOD03750
CC                              HARRIER G                               MOD03760
C                                                                       MOD03770
C       HARRIER G IS THE CODE USED TO GENERATE THE A,B AND C            MOD03780
C       MATRICIES FOR THE COVARIANCE CALCULATION OF THE HARRIER         MOD03790
C       LONGITUDNAL AXIS SRFIMF CONTROL WITH A GUST INPUT.              MOD03800
C                                                                       MOD03810
C*****************************************************************     MOD03820
        DIMENSION B(10,4),H(3,10),TR(4)                                 MOD03830
        REAL K,KX,KXD                                                   MOD03840
        COMMON BP,CP,K,KX,KXD,TOU,CPG,N,PD(100),Q(10,10),A(10,10)       MOD03850
        WRITE(6,100)                                                    MOD03860
100     FORMAT(//,20X,90('*'),//,40X,'HARRIER MODEL ',                  MOD03870
```

```
      1' ALL QUANTITIES MEASURED',//,20X,90('*'),//)                    MOD03880
       N=10                                                             MOD03890
       L=4                                                              MOD03900
       M=3                                                              MOD03910
       K=20.                                                            MOD03920
       KX=4.                                                            MOD03930
       KXD=3.                                                           MOD03940
       TOU=.1                                                           MOD03950
       SIGMA1=1.                                                        MOD03960
       SIGMA2=1.                                                        MOD03970
       SIGMA3=1.                                                        MOD03980
       SIGMA4=1.                                                        MOD03990
       TR(1)=10.                                                        MOD04000
       TR(2)=10.                                                        MOD04010
       TR(3)=10.                                                        MOD04020
       TR(4)=6.5                                                        MOD04030
C                                                                       MOD04040
CC     CALCULATE THE A MATRIX                                           MOD04050
C                                                                       MOD04060
       DO 1 I=1,N                                                       MOD04070
       DO 1 J=1,N                                                       MOD04080
     1 A(I,J)=0.0                                                       MOD04090
       A(1,2)=1.                                                        MOD04100
       A(1,10)=-.0026                                                   MOD04110
       A(2,3)=1.                                                        MOD04120
       A(2,5)=.25                                                       MOD04130
       A(2,10)=-.00473                                                  MOD04140
       A(3,4)=1.                                                        MOD04150
       A(3,5)=-.06115                                                   MOD04160
       A(3,10)=.00306                                                   MOD04170
       A(4,1)=-.00748                                                   MOD04180
       A(4,2)=-.0675                                                    MOD04190
       A(4,3)=.45                                                       MOD04200
       A(4,4)=-.4906                                                    MOD04210
       A(4,5)=.14428                                                    MOD04220
       A(4,10)=.00346                                                   MOD04230
       A(5,1)=-K*KX/TOU                                                 MOD04240
       A(5,2)=-K*KXD/TOU                                                MOD04250
       A(5,3)=-K/TOU                                                    MOD04260
       A(5,5)=-(.25*K+1.)/TOU                                           MOD04270
       A(5,6)=K/TOU                                                     MOD04280
       A(5,7)=-K*KXD/TOU                                                MOD04290
       A(5,8)=-K/TOU                                                    MOD04300
       A(5,9)=-K*KX/TOU                                                 MOD04310
       A(5,10)=(KXD*.0026+.00473)*K/TOU                                 MOD04320
       A(6,1)=-KX/TOU                                                   MOD04330
       A(6,2)=-KXD/TOU                                                  MOD04340
       A(6,3)=-1./TOU                                                   MOD04350
```

```
      A(6,5)=-.25/TOU                                                   MOD04360
      A(6,7)=-KXD/TOU                                                   MOD04370
      A(6,8)=-1./TOU                                                    MOD04380
      A(6,9)=-KX/TOU                                                    MOD04390
      A(6,10)=A(5,10)/K                                                 MOD04400
      A(7,7)=-1./TR(1)                                                  MOD04410
      A(8,8)=-1./TR(2)                                                  MOD04420
      A(9,9)=-1./TR(3)                                                  MOD04430
      A(10,10)=-1./TR(4)                                                MOD04440
C                                                                       MOD04450
CC    B MATRIX                                                          MOD04460
C                                                                       MOD04470
      DO 2 I=1,N                                                        MOD04480
      DO 2 J=1,L                                                        MOD04490
    2 B(I,J)=0.0                                                        MOD04500
      B(7,1)=1.                                                         MOD04510
      B(8,2)=1.                                                         MOD04520
      B(9,3)=1.                                                         MOD04530
      B(10,4)=1.                                                        MOD04540
C                                                                       MOD04550
CC    Q MATRIX                                                          MOD04560
C                                                                       MOD04570
      DO 3 I=1,L                                                        MOD04580
      DO 3 J=1,L                                                        MOD04590
    3 Q(I,J)=0.0                                                        MOD04600
      Q(1,1)=(2./TR(1))*SIGMA1**2                                       MOD04610
      Q(2,2)=(2./TR(2))*SIGMA2**2                                       MOD04620
      Q(3,3)=(2./TR(3))*SIGMA3**2                                       MOD04630
      Q(4,4)=(2./TR(4))*SIGMA4**2                                       MOD04640
C                                                                       MOD04650
CC    C MATRIX                                                          MOD04660
C                                                                       MOD04670
      DO 4 I=1,M                                                        MOD04680
      DO 4 J=1,N                                                        MOD04690
    4 H(I,J)=0.0                                                        MOD04700
      H(1,1)=1.0                                                        MOD04710
      H(2,2)=1.0                                                        MOD04720
      H(3,3)=1.0                                                        MOD04730
      NPT=60                                                            MOD04740
      T1=0.                                                             MOD04750
      T2=60.                                                            MOD04760
      CALL ECHO(A,B,Q,H,TR,N,M,L)                                       MOD04770
      CALL MODAL(A,B,H,N,M,L)                                           MOD04780
      CALL VARY (B,L,NPT,T1,T2)                                         MOD04790
      STOP                                                              MOD04800
      END                                                               MOD04810
```

```
C*************************************************************************  MOD00040
C                                                                          MOD00050
C                                                                          MOD00060
C                    SJBROUTINE MODAL (A,B,H,N,M,L)                        MOD00070
C                                                                          MOD00080
C     THIS SUBROUTINE IS USED TO TRANSFORM A SYSTEM OF THE FORM            MOD00090
C                                                                          MOD00100
C              XDOT=AX+BU                                                  MOD00110
C                                                                          MOD00120
C              Y=CX                                                        MOD00130
C                                                                          MOD00140
C     TO MODAL COORDINATES.                                                MOD00150
C                                                                          MOD00160
C                                                                          MOD00170
C*************************************************************************  MOD00180
      SUBROUTINE MODAL(A,B,H,N,M,L)                                        MOD00190
      DIMENSION A(N,N),B(N,L),H(M,N),AC(10,10),C(10,10),D(10,10)           MOD00200
     1,G(4,10),T(10,10)                                                    MOD00210
      COMPLEX Z(10,10),W(10),WK(99),ENORM,VNORM                            MOD00220
      REAL K,KX,KXD                                                        MOD00230
      DO 1 I=1,N                                                           MOD00240
      DO 1 J=1,N                                                           MOD00250
1     AC(I,J)=A(I,J)                                                       MOD00260
      WRITE(6,210)                                                         MOD00270
210   FORMAT(//,20X,70('*'),/,20X,'*',/,20X,'*      MODAL ANALYSIS         MOD00280
     1FOR THE SYSTEM',/,20X,'*',/,20X,70('*'),//)                          MOD00290
      IA=N                                                                 MOD00300
      IZ=N                                                                 MOD00310
      IJOB=1                                                               MOD00320
      CALL EIGRF(A,N,N,IJOB,W,Z,IZ,WK,IER)                                 MOD00330
C                                                                          MOD00340
C     WRITE OUT THE EIGENCVALUES                                           MOD00350
C                                                                          MOD00360
      WRITE(6,201)                                                         MOD00370
201   FORMAT(//,10X,'THE EIGENVALUES ARE',//)                              MOD00380
      WRITE(6,101) (W(I),I=1,N)                                            MOD00390
101   FORMAT(1X,16F6.2,/)                                                  MOD00400
C                                                                          MOD00410
C     WRITE THE EIGENVECTORS_                                              MOD00420
C                                                                          MOD00430
      WRITE(6,202)                                                         MOD00440
202   FORMAT(//,10X,'THE EIGENVECTORS ARE',//)                             MOD00450
      WRITE(6,101) ((Z(I,J),J=1,N),I=1,N)                                  MOD00460
C                                                                          MOD00470
C     EIGENVECTOR NORMALIZATION                                            MOD00480
C                                                                          MOD00490
      J=1                                                                  MOD00500
10    CONTINUE                                                             MOD00510
```

```
      FNORM=0.0                                                          MOD00520
      IF(AIMAG(W(J)).NE.0) GO TO 4                                       MOD00530
C                                                                        MOD00540
CC    EIGENVECTOR IS REAL                                                MOD00550
C                                                                        MOD00560
      DO 2 I=1,N                                                         MOD00570
2     FNORM=FNORM+REAL(Z(I,J)**2)                                        MOD00580
      DO 3 I=1,N                                                         MOD00590
3     T(I,J)=REAL(Z(I,J))/SQRT(FNORM)                                    MOD00600
      GO TO 7                                                            MOD00610
C                                                                        MOD00620
CC    COMPLEX NORMALIZATION                                              MOD00630
CC                                                                       MOD00640
4     DO 5 I=1,N                                                         MOD00650
      TAT=REAL(Z(I,J))**2                                                MOD00660
      IF(TAT.LE.FNORM) GO TO 5                                           MOD00670
      IMAX=I                                                             MOD00680
      FNORM=TAT                                                          MOD00690
5     CONTINUE                                                           MOD00700
CCC                                                                      MOD00710
CC    THE I'TH ROW WILL BE NORMALIZED TO 1+0I                            MOD00720
C                                                                        MOD00730
      XN1=REAL(Z(IMAX,J))                                                MOD00740
      XN2=AIMAG(Z(IMAX,J))                                               MOD00750
      ENORM=CMPLX(XN1/(XN1+XN2**2),-XN2/(XN1+XN2**2))                    MOD00760
      DO 6 I=1,N                                                         MOD00770
      VNORM=Z(I,J)*ENORM                                                 MOD00780
      T(I,J)=REAL(VNORM)                                                 MOD00790
6     T(I,J+1)=AIMAG(VNORM)                                              MOD00800
      J=J+1                                                              MOD00810
7     J=J+1                                                              MOD00820
      IF(J.LE.N) GO TO 10                                                MOD00830
C                                                                        MOD00840
CC    WRITE THE NORMALIZED MATRIX                                        MOD00850
C                                                                        MOD00860
      WRITE(5,203)                                                       MOD00870
203   FORMAT(//,10X,'THE NORMALIZED TRANSFER MATRIX IS',//)              MOD00880
      WRITE(6,100) ((T(I,J),J=1,N),I=1,N)                                MOD00890
C                                                                        MOD00900
CC    INVERT T                                                           MOD00910
C                                                                        MOD00920
      IDGT=0                                                             MOD00930
      CALL LINV2F(T,N,IA,A,IDGT,WK,IER)                                  MOD00940
      WRITE(6,204)                                                       MOD00950
204   FORMAT(//,10X,'THE INVERSE OF THE TRANSFORM IS',//)                MOD00960
      WRITE(6,100) ((A(I,J),J=1,N),I=1,N)                                MOD00970
C                                                                        MOD00980
C     TAKE T INV*A*T                                                     MOD00990
```

```
C                                                                       MOD01000
      CALL VMULFF(A,AC,N,N,N,N,IA,C,N,IER)                              MOD01010
      CALL VMULFF(C,T,N,N,N,IA,IA,AC,N,IER)                             MOD01020
      WRITE(6,205)                                                      MOD01030
205   FORMAT(//,10X,'THE DIAGINAL MATRIX IS',//)                        MOD01040
      WRITE(6,100) ((AC(I,J),J=1,N),I=1,N)                              MOD01050
C                                                                       MOD01060
C     CALCULATE THE CONTROL MATRIX                                      MOD01070
C                                                                       MOD01080
      WRITE (6,206)                                                     MOD01090
206   FORMAT(//,10X,'THE CONTROL MATRIX IS',//)                         MOD01100
      CALL VMULFF(A,B,N,N,L,N,N,D,N,IER)                                MOD01110
      DO 30 I=1,N                                                       MOD01120
30    WRITE(6,104) (D(I,J),J=1,L)                                       MOD01130
104   FORMAT(/,8(2X,F7.2,2X))                                           MOD01140
C                                                                       MOD01150
C     CALCULATE Y=CT                                                    MOD01160
C                                                                       MOD01170
      WRITE(6,207)                                                      MOD01180
207   FORMAT(//,10X,'THE MEASUREMENT VECTOR IS',//)                     MOD01190
      CALL VMULFF(H,T,M,N,N,M,IA,G,M,IER)                               MOD01200
      WRITE(6,100) ((G(I,J),J=1,N),I=1,M)                               MOD01210
100   FORMAT(/,10(2X,F7.2))                                             MOD01220
      RETURN                                                            MOD01230
      END                                                               MOD01240
                                                                        MOD01250
                                                                        MOD01260
                                                                        MOD01270
                                                                        MOD01280
C***********************************************************************MOD01290
C                                                                       MOD01300
C                  SUBROUTINE VARY (B,L,NPT,T1,T2)                      MOD01310
C                                                                       MOD01320
C     VARY IS USED TO CALCULATE THE COVARIANCE OF THE STATES OF         MOD01330
C     A SYSTEM.THE COVARIANCE IS FOUND BY SOLVING THE LYAPUNOV          MOD01340
C     DIFFERENTIAL EQUATION USING THE ISML SUBROUTINE DVERK.            MOD01350
C                                                                       MOD01360
C***********************************************************************MOD01370
      SUBROUTINE VARY (B,L,NPT,T1,T2)                                   MOD01380
      DIMENSION B(10,4),P(100),PDOT(100),D(10,10),W(100.10),            MOD01390
     1G(10,10),C(24)                                                    MOD01400
      EXTERNAL FCN                                                      MOD01410
      COMMON BP,CP,K,KX,KXD,TOU,CPG,N,PD(100),Q(10,10),A(10,10)         MOD01420
      WRITE(6,200)                                                      MOD01430
200   FORMAT(//,20X,90('*'),//,40X,'VARIANCE ANALYSIS',                 MOD01440
     1//,20X,90('*'),/)                                                 MOD01450
C                                                                       MOD01460
C     START CLACULATIONS                                                MOD01470
```

```
C                                                                       MOD01480
      TST=.00005                                                        MOD01490
      NSQ=N*N                                                           MOD01500
      NW=NSQ                                                            MOD01510
      TIME=T1                                                           MOD01520
      TOL=.001                                                          MOD01530
      XNPT=NPT                                                          MOD01540
      STEP=(T2-T1)/XNPT                                                 MOD01550
      DO 1 I=1,N                                                        MOD01560
      DO 1 J=1,N                                                        MOD01570
      G(I,J)=0.0                                                        MOD01580
      DO 1 K=1,L                                                        MOD01590
    1 G(I,J)=G(I,J)+B(I,K)*Q(K,J)                                       MOD01600
CC                                                                      MOD01610
CC    G=B*QA                                                            MOD01620
C                                                                       MOD01630
      DO 2 I=1,N                                                        MOD01640
      DO 2 J=1,N                                                        MOD01650
      Q(I,J)=0.0                                                        MOD01660
      DO 2 K=1,L                                                        MOD01670
    2 Q(I,J)=Q(I,J)+G(I,K)*B(J,K)                                       MOD01680
CC                                                                      MOD01690
CC    Q=B*Q*BT                                                          MOD01700
CC                                                                      MOD01710
CC    NOW SET INITIAL VALUES OF P                                       MOD01720
C                                                                       MOD01730
      DO 3 I=1,N                                                        MOD01740
      DO 3 J=1,N                                                        MOD01750
    3 G(I,J)=0.0                                                        MOD01760
CC                                                                      MOD01770
CC    G IS THE MATRIX OF THE P VECTOR SET TO 0.0                        MOD01780
CC                                                                      MOD01790
CC    SET UP TO CALL DVERK                                              MOD01800
C                                                                       MOD01810
      DO 4 I=1,24                                                       MOD01820
    4 C(I)=0.0                                                          MOD01830
      WRITE (6,700) TIME                                                MOD01840
  700 FORMAT(/,10X,'AT TIME= ',F5.2,/)                                  MOD01850
      CALL MTV (N,NSQ,G,P)                                              MOD01860
   20 TIMEE=TIME+STEP                                                   MOD01870
      IND=1                                                             MOD01880
      CALL DVERK (NSQ,FCN,TIME,P,TIMEE,TOL,IND,C,NW,W,IER)              MOD01890
      WRITE(6,750) IND                                                  MOD01900
  750 FORMAT(10X,'IND=',I3,' AFTER CALL TO DVERK')                      MOD01910
      CALL VTM (N,NSQ,G,P)                                              MOD01920
      WRITE (6,700) TIME                                                MOD01930
      WRITE(6,800)                                                      MOD01940
  800 FORMAT(//,20X,'THE P MATRIX IS',//)                               MOD01950
```

```
      WRITE(6,100) ((G(I,J),J=1,N),I=1,N)                                MOD01960
      X5=SQRT(G(5,5))                                                    MOD01970
      CALL VTM (N,NSQ,D,PD)                                              MOD01980
C     WRITE(6,801)                                                       MOD01990
801   FORMAT(//,20X,'P DOT IS',//)                                       MOD02000
C     WRITE(6,100) ((D(I,J),J=1,N),I=1,N)                                MOD02010
      SUM=0.0                                                            MOD02020
      DO 5 I=1,NSQ                                                       MOD02030
5     SUM=SUM+ABS(PD(I))                                                 MOD02040
      WRITE(6,802) SUM                                                   MOD02050
802   FORMAT(10X,'THE SUM OF THE P DOT TERMS IS ',E15.5)                 MOD02060
      IF(SUM.LE.TST) GO TO 50                                            MOD02070
      IF(TIMEE.GE.T2) GO TO 51                                           MOD02080
      GO TO 20                                                           MOD02090
50    WRITE(6,710)                                                       MOD02100
710   FORMAT(//,20X,'THE LAST P IS A STEADY STATE',//)                   MOD02110
      RETURN                                                             MOD02120
51    CALL VTM (N,NSQ,G,PD)                                              MOD02130
      WRITE (6,720)                                                      MOD02140
720   FORMAT(//,20X,'END OF RUN THE FINAL PDOT IS',//)                   MOD02150
      WRITE(6,100) ((G(I,J),J=1,N),I=1,N)                                MOD02160
100   FORMAT(10(2X,E10.4),/)                                             MOD02170
      RETURN                                                             MOD02180
      END                                                                MOD02190
C                                                                        MOD02200
CC                SUBROJTINE FCN (NSQ,TIME,P,PDOT)                       MOD02210
CC                                                                       MOD02220
CC    FCN IS USED BY DVERK.IT PROVIDES THE FORM OF THE                   MOD02230
CC    DIFFERENTIAL EQUATION.                                             MOD02240
C                                                                        MOD02250
      SUBROUTINE FCN (NSQ,TIME,P,PDOT)                                   MOD02260
      DIMENSION D(10,10),G(10,10),P(NSQ),PDOT(NSQ)                       MOD02270
      COMMON BP,CP,K,KX,KXD,TOU,CPG,N,PD(100),Q(10,10),A(10,10)          MOD02280
      CALL VTM(N,NSQ,G,P)                                                MOD02290
      DO 1 I=1,N                                                         MOD02300
      DO 1 J=1,N                                                         MOD02310
      D(I,J)=Q(I,J)                                                      MOD02320
      DO 1 K=1,N                                                         MOD02330
1     D(I,J)=D(I,J)+A(I,K)*G(K,J)+G(I,K)*A(J,K)                          MOD02340
      CALL MTV (N,NSQ,D,PDOT)                                            MOD02350
      DO 2 I=1,NSQ                                                       MOD02360
2     PD(I)=PDOT(I)                                                      MOD02370
      RETURN                                                             MOD02380
      END                                                                MOD02390
C                                                                        MOD02400
CC    SUBROUTINES MTV AND VTM                                            MOD02410
C                                                                        MOD02420
      SUBROUTINE MTV (N,NSQ,A,B)                                         MOD02430
```

```
      DIMENSION A(N,N),B(NSQ)                                          MOD02440
      K=1                                                              MOD02450
      DO 1 I=1,N                                                       MOD02460
      DO 1 J=1,N                                                       MOD02470
      B(K)=A(I,J)                                                      MOD02480
    1 K=K+1                                                            MOD02490
      DEBUG SUBCHK                                                     MOD02500
      RETURN                                                           MOD02510
      END                                                              MOD02520
C                                                                      MOD02530
      SUBROUTINE VTM (N,NSQ,A,B)                                       MOD02540
      DIMENSION A(N,N),B(NSQ)                                          MOD02550
      K=1                                                              MOD02560
      DO 1 I=1,N                                                       MOD02570
      DO 1 J=1,N                                                       MOD02580
      A(I,J)=B(K)                                                      MOD02590
    1 K=K+1                                                            MOD02600
      DEBUG SUBCHK                                                     MOD02610
      RETURN                                                           MOD02620
      END                                                              MOD02630
```

```
*****************************************************************
*
*
*      **    HARRIER LONDITUDNAL SRFIMF CONTROL SIMULATION   **
*
*****************************************************************
// EXEC CSMPXV
//X.COMPRINT DD DUMMY
//X.SYSPRINT DD DUMMY
//X.SYSIN DD *
*
*      SIMULATION OF OPEN LOOP PLANT
*
X1P=INTGRL(0.0,X2P)
X2P=INTGRL(0.0,X3P)
X3P=INTGRL(0.0,X4P)
X4P=INTGRL(0.0,X4PDOT)
X4PDOT=.25*XC-.49*X4P+.45*X3P-.0665*X2P-.00748*X1P
ACCELP=-.00189*X1P-.016625*X2P+.1144*X3P-.061*X4P+.25*XC
POSITP=.00189*X1P+.0615*X2P+.25*X3P
VELP=.00189*X2P+.0615*X3P+.25*X4P
*
*
*      CLOSED LOOP SYSTEM SIMULATION
*
       X1=INTGRL(0.0,X2)
       X2=INTGRL(0.0,X3)
       X3=INTGRL(0.0,X4)
       X4=INTGRL(0.0,X4DOT)
       X4DOT=X5-.49*X4+.45*X3-.0665*X2-.00748*X1
       ACCEL=-.00189*X1-.016625*X2+.1144*X3-.061*X4+.25*X5
       POSIT=.00189*X1+.0615*X2+.25*X3
       VEL=.00189*X2+.0615*X3+.25*X4
       X5=REALPL(0.0,TOU,IS)
       XC=STEP(0.0)
       WS=-KXD*VX+KX*(XC-X)+VS
       IS=K*WS
       VS=AS-AX
       AS=REALPL(0.0,TOU,WS)
       AX=ACCEL
       VX=VEL
       X=POSIT
       K=100.
       KX=4.
       KXD=3.
       TOU=.1
TIMER FINTIM=10,OUTDEL=.2
OUTPUT POSIT,POSITP
OUTPUT TIME,POSIT,POSITP
PAGE XYPLOT
       END
       STOP
```

APPENDIX B

TABULATED DATA

Time (sec)	σ_{1p}	σ_{1v}	σ_{1a}	σ_{2p}	σ_{2v}	σ_{2a}	σ_{3p}	σ_{3v}	σ_{3a}
0	0	0	0	0	0	0	0	0	0
.5	.045	.033	.011	.191	.149	.050	.764	.573	.191
1.0	.164	.123	.074	.315	.236	.079	.825	.619	.206
2.0	.410	.307	.102	.342	.256	.085	1.022	.767	.256
3.0	.564	.423	.141	.344	.258	.086	1.283	.962	.321
4.0	.660	.495	.165	.347	.260	.087	1.475	1.106	.369
5.0	.728	.546	.182	.348	.261	.087	1.613	1.210	.403
6.0	.780	.585	.195	.349	.262	.087	1.717	1.288	.429
7.0	.819	.615	.205	.350	.262	.087	1.797	1.348	.449
8.0	.851	.638	.213	.351	.263	.088	1.861	1.395	.465
9.0	.875	.657	.219	.351	.263	.088	1.911	1.433	.478
10.0	.895	.671	.224	.352	.264	.088	1.950	1.464	.488
20.0	.968	.726	.242	.354	.265	.088	2.100	1.576	.525
40.0	.979	.734	.245	.354	.265	.089	2.123	1.592	.531
60.0	.979	.734	.245	.354	.266	.089	2.123	1.592	.531
S.S.	.979	.734	.245	.354	.266	.089	2.123	1.592	.531

Table B-1 Tabulated Data
for figures 4-4 through 4-6.

Time (sec)	σ_{1a}	σ_{2a}	σ_{3a}
0	0	0	0
2.0	.530	.675	2.005
4.0	2.858	1.781	7.745
6.0	8.070	3.060	18.515
8.0	14.649	4.412	34.742
10.0	24.399	5.791	56.569

The rate of change in P, $\dot{P}$ after 10 seconds was

	$\sigma_{\dot{P}1}$	$\sigma_{\dot{P}2}$	$\sigma_{\dot{P}3}$
10.0	16.422	2.826	37.350

Table B-2
Tabulated data for figure 4-7.

Time (sec)	σ_{1p}	σ_{1v}	σ_{2p}	σ_{2v}	σ_{3p}	σ_{3v}
0	0	0	0	0	0	0
.5	.020	.026	.091	.119	.351	.458
1.0	.076	.099	.149	.189	.379	.495
2.0	.189	.246	.157	.205	.470	.613
3.0	.259	.338	.158	.206	.590	.770
4.0	.304	.396	.160	.208	.679	.885
5.0	.335	.437	.160	.209	.740	.968
6.0	.359	.468	.160	.209	.790	1.030
7.0	.377	.492	.161	.210	.827	1.078
8.0	.391	.510	.161	.210	.856	1.116
9.0	.403	.525	.162	.211	.879	1.147
10.0	.412	.537	.162	.211	.898	1.171
20.0	.445	.581	.163	.212	.966	1.261
S.S	.450	.590	.163	.212	.970	1.300

Table B-3 Tabulated data for figures 4-9 through 4-11.

Time (sec)	σ_{1p}	σ_{1v}	σ_{1a}	σ_{2p}	σ_{2v}	σ_{2a}	σ_{5p}	σ_{5v}	σ_{5a}
0	0	0	0	0	0	0	0	0	0
.5	.440	.081	.011	.201	.210	.050	2.672	2.010	.668
1.0	.166	.125	.042	.319	.239	.080	2.807	2.106	.702
2.0	.413	.310	.103	.345	.259	.086	3.237	2.428	.809
3.0	.566	.425	.142	.348	.261	.087	3.286	2.465	.822
4.0	.662	.496	.165	.350	.263	.088	3.288	2.466	.822
5.0	.730	.547	.182	.351	.264	.088	3.291	2.468	.822
6.0	.781	.586	.195	.353	.264	.088	3.300	2.475	.825
7.0	.820	.615	.205	.353	.265	.088	3.320	2.490	.830
8.0	.852	.639	.213	.354	.266	.089	3.345	2.511	.837
9.0	.876	.657	.219	.355	.266	.089	3.384	2.538	.846
10.0	.896	.672	.224	.355	.266	.089	3.425	2.570	.856
20.0	.969	.727	.242	.357	.268	.089	3.821	2.866	.955
S.S.	.980	.734	.245	.360	.270	.089	3.900	2.900	.970

Table B-4 Tabulated data for figures 5-4 through 5-6.

Time (sec)	σ_{1g}	σ_{2g}	σ_{5g}	σ_{acc_g}	σ_g
0	0	0	0	0	0
1.0	.0013	.006	.076	.013	2.573
2.0	.0014	.009	.110	.040	3.390
3.0	.0014	.011	.227	.102	3.882
4.0	.0015	.011	.402	.191	4.207
5.0	.0015	.012	.622	.301	4.431
6.0	.0015	.012	.874	.427	4.588
7.0	.0016	.013	1.150	.564	4.701
8.0	.0016	.013	1.442	.701	4.782
9.0	.0016	.013	1.743	.860	4.840
10.0	.0016	.013	2.050	1.013	4.884
20.0	.0016	.013	4.750	2.360	4.959
40.0	.0016	.013	7.169	3.567	5.00
60.0	.0017	.013	7.783	3.874	5.00
S.S.	.0017	.013	7.800	4.00	4.00

Table B-5 Tabulated data for figure 5-7.

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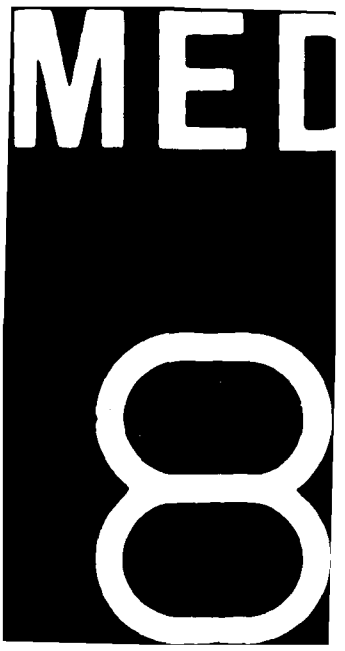
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